8

Markov chain simulation to evaluate user-defined management strategies

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Objectives

From this chapter the reader should gain knowledge of:

- the characteristics of Markov chains
- the concepts and definitions of states
- the long-run properties of Markov chains

The method is introduced by two simplified examples and further illustrated with an application, simulating herd dynamics in sow herds in order to evaluate the effects of different management strategies on the technical and economic results of the herd.

8.1 Introduction

Markov chains are used to model the evolution of systems or processes over repeated trials or successive time periods. In animal health economics, Markov chain simulation has been used most extensively to evaluate the impact of alternative control strategies on the spread of disease (Carpenter, 1988; Dijkhuizen, 1989). Dynamic programming is also an application of Markov chains and is often used to determine the optimal insemination and replacement decisions for individual cows and sows (see Chapter 7).

Key issue of interest in Markov chain models is the study of events and sequential decision making under uncertainty. Intervals of time separate the stages at which events occur and decisions can be made, and the effect of a decision at any stage is to influence the transition from the current and succeeding state. Central to the theory of Markov chain models are the concepts of states and transitions. The distribution of the system or process over states at a certain moment can be derived from the distribution at the moment before and the transitions possible for each state. Characteristic is the Markovian property, implying that a transition from state i to state j depends only on the state currently occupied (Hillier & Lieberman, 1990).

8.2 Markov chains in general

A Markov chain model has two components: **states** and **transitions**. The Markov chain represents a system or process that moves between a number of states. The states may constitute a qualitative as well as a quantitative characterization of the system. If the present state is $S_n = i$, then there is a certain probability that the next state visited is $S_{n+1} = j$. This probability does not depend on the other states visited prior to entry into state i. In other words, the conditional probability of any future event, given any past event and the present state $S_n = i$, is independent of the past event and depends only upon the present state of the process. This is referred to as the **Markovian property** (Hillier & Lieberman, 1990). This principle can be illustrated with an example of a frog in a lily pond (Howard, 1971). The frog in this example always sits on a pad; it never swims in the water. From time to time the frog jumps into the air and lands on the same lily pad or on a different one. We are interested in the location of the frog after successive jumps. Let us assume the pond to have a finite number of lily pads, numbered from 1 to N, here from 1 to 3 (see Figure 8.1). The lily pads the frog can sit on represent the states in the Markov chain. The 'process' that moves from state to state is the frog. The 'transitions' are the jumps of the frog. If this situation is modelled as a Markov chain it means that the probability that the frog will jump from lily pad 1 to lily pad 2 only depends on the current state (lily pad 1) of the frog. The probability is independent of the lily pads that the frog occupied before it was on lily pad 1. The conditional probabilities $P\{S_{n+1} = j \mid S_n = i\}$ are called transition probabilities and are usually denoted by p_{ij}. If the transition probabilities p_{ij} are constant over time, they are stationary. Moreover, a Markov chain has a finite number of states and a discrete time parameter. Due to the Markovian property, a Markov chain is said to exhibit a lack of memory. However, by adding more states **memory** can be introduced into the model. If, for instance, in the frog example, the probability is dependent on the present pad occupied and on the one before that, the states will have to be reorganized. In that case we will have N^2 states, each representing a combination of two pads possible, representing pads occupied (see Figure 8.2).

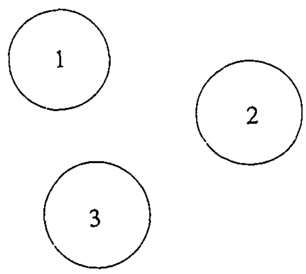


Figure 8.1 The lily pond

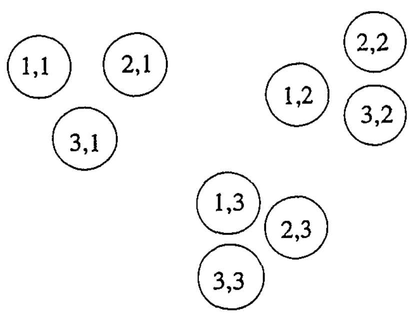


Figure 8.2 The lily pond with memory; (i,j); i =1 ... 3, previous pad; j = 1 ... 3, present pad)

The distribution over states can be represented by a state vector X. A convenient notation for representing the transition probabilities is the transition matrix P, with elements p_{ij}. If the Markov chain consists of three states, the transition matrix is as follows:

$$P = \begin{bmatrix} p_{11} & p_{12} & p_{12} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix}$$

p_{13} represents the probability that the process which is in state 1 during period n will move to state 3 in period n+1. Each p_{ij} is a probability, and thus $0 \leq p_{ij} \leq 1$, for $1 \leq i, j \leq M$ (M equals the total number of states). The system or process must be in one of the m states after the transition from any state i and thus $\Sigma_j p_{ij} = 1$ for all i.
The state vector at time n+1, X_{n+1}, can be derived from the state vector at time n, X_n, and the transition matrix P: $X_{n+1} = X_n$ P. If transition probabilities are stationary, $X_{n+1} = X_0 P^{(n)}$. $P^{(n)}$ is denoted as the matrix of n-step transition probabilities. The elements of $P^{(n)}$, $p_{ij}^{(n)}$, are the conditional probabilities that the system or process, starting in state i, will be in state j after exactly n time steps. The n-step transition probability matrix can be obtained by computing the n^{th} power of the one-step transition matrix. So, $P^{(n)} = P\,P\,P\,P \ldots P = P^n$. So in fact, the initial state vector X_0 and the transition matrix P determine the state vector at each following moment. The characteristics of Markov chains will be illustrated with two simplified examples.

Sow replacement

In the first example, sow herd dynamics is modelled for a herd with a constant number of sows. After weaning litter i, a sow may be culled and replaced with a young sow that is about to have her first litter, or is retained and will produce the next litter. In the Markov chain the number of states is restricted to 3: (1) sow having litter 1, (2) sow having litter 2, and (3) sow having litter 3 or higher. The possible transitions for the Markov chain are from $S_n = i$ to $S_{n+1} = i+1$, when the sow is retained after weaning. If the sow is culled and replaced, the transition from $S_n = i$ to $S_{n+1} = 1$ will take place. All other transitions have probability zero. The non-zero transition probabilities are estimated from available data on several herds (Table 8.1).

Table 8.1 Data on number of sows culled and retained

Litter	Number of sows		Relative frequency	
number	culled	retained	culled	retained
1	216	864	0.2	0.8
2	184	552	0.25	0.75
≥3	614	1432	0.3	0.7

Thus, p_{11}, the probability that a sow that had litter 1 is culled equals 0.20. The resulting transition matrix is as follows:

$$P = \begin{bmatrix} 0.2 & 0.8 & 0 \\ 0.25 & 0 & 0.75 \\ 0.3 & 0 & 0.7 \end{bmatrix}$$

From P it is possible to derive $P^{(2)}$ by multiplying the one-step transition probability matrix by itself, $P^{(2)} = P^2 = P\,P$.

$$P^{(2)} = \begin{bmatrix} 0.24 & 0.16 & 0.6 \\ 0.275 & 0.2 & 0.525 \\ 0.27 & 0.24 & 0.49 \end{bmatrix}$$

$p_{11}^{(2)}$, one of the elements of $P^{(2)}$ is calculated as 0.2 x 0.2 + 0.8 x 0.25 + 0 x 0.3 = 0.24. $P^{(4)} = P^4 = P^2\,P^2$ and is given below:

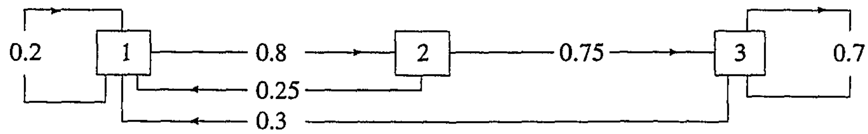
$$P^{(4)} = \begin{bmatrix} 0.264 & 0.214 & 0.522 \\ 0.263 & 0.210 & 0.527 \\ 0.263 & 0.209 & 0.528 \end{bmatrix}$$

The transition matrices presented above are used to derive the state vector at n=1, n=2 and n=4. In the initial state vector X_0, all 100 animals in the herd are sows that are about to have the first litter (X_0 = {100,0,0}). The state vectors at different time periods are given in Table 8.2.

Table 8.2 State vector X at different time periods n (X0Pn)

State	**State vector**			
	X_0	X_1	X_2	X_4
1	100	20	24	26.4
2	0	80	16	21.4
3	0	0	60	54.0

Another type of representation of a Markov chain is a transition diagram, which, for this example, is as follows:



Spread of a disease

In the second example, the spread of a certain disease among animals in a herd is modelled. Susceptible animals at time n can become infected at n+1 with a probability of 0.40. The infected animals in time period n+1 become immune at time period n+2 (probability 0.80), or die owing to the disease in time period n+2 (probability 0.20). The Markov chain consists of four states, (1) uninfected animals, (2) infected animals, (3) animals immune after infection, and (4) animals that died after infection. The transition matrix P is as follows:

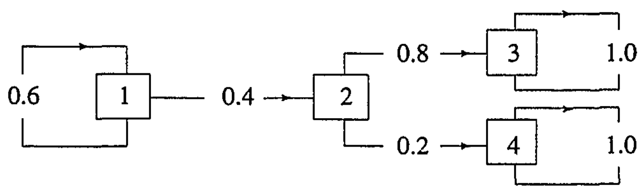
$$P = \begin{bmatrix} 0.6 & 0.4 & 0 & 0 \\ 0 & 0 & 0.8 & 0.2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

From P, $P^{(2)}$ and $P^{(4)}$ can be derived in the same way as was done in the first example.

$$P^{(2)} = P^2 = P\,P = \begin{bmatrix} 0.36 & 0.24 & 0.32 & 0.08 \\ 0 & 0 & 0.8 & 0.2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P^{(4)} = P^4 = P^2\,P^2 = \begin{bmatrix} 0.130 & 0.086 & 0.627 & 0.157 \\ 0 & 0 & 0.8 & 0.2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

A graphical representation of the Markov chain can be produced by drawing the transition diagram:



8.3 Concepts and definitions of states

The transition probabilities associated with the states play an important role in the study of the system modelled by the Markov chain. Before describing the special properties of Markov chains, some concepts and definitions concerning states are presented.

State j is said to be **accessible** from state i if $p_{ij}^{(n)} > 0$ for some $n \geq 0$. All states of a Markov chain are accessible when there is a value of n for which $p_{ij}^{(n)} > 0$ for all i and j. In the first example, all states are accessible, since $p_{ij}^{(2)} > 0$ for all i and j. In the second example, state 1 is not accessible from state 2, as can easily be derived from the transition diagram. Also, in the n-step transition matrix, element $p_{21}^{(n)}$ equals 0 for all n. However, state 2 is accessible from state 1.

States i and j are said to **communicate** when state j is accessible from state i, and state i is accessible from state j. In the first example, all states communicate; in the second example, none of them do. In general:

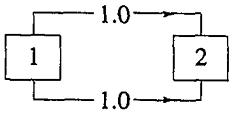
- any state i communicates with state i, since $p_{ii}^{(0)} = 1$;
- if state i communicates with state j, then state j communicates with state i; and
- if state i communicates with state j, and state j communicates with state k, then state i communicates with state k.

The states of a Markov chain can be divided into one or more disjoint **classes**. Two states that communicate always belong to the same class. A class may consist of a single state only. If all states in a Markov chain communicate, as in the first example, there is only one class and such a Markov chain is said to be irreducible. The second example contains 4 classes; all states form a separate class.

States differ in the probability whether or not a process, starting in state i, will ever return to state i. This probability is denoted by f_{ii}. For a **recurrent** state, f_{ii} equals 1. If $f_{ii} < 1$, a state is called **transient**. A special case of a recurrent state is an **absorbing** state. For an absorbing state, the one-step transition probability p_{ii} equals 1; once a process enters the state, it cannot leave it again.

All states in a class are either recurrent or transient, and therefore a class is denoted as recurrent or transient. Each finite-state Markov chain consists of at least one recurrent class of states. The first example consists of one recurrent class of states. The second example has two transients classes (1 and 2) and two recurrent classes (3 and 4). Once a recurrent class is entered, the process will never leave it again. Once a process leaves a transient class, the process will never enter it again. This can easily be inferred from the transition diagram in the second example.

The **period** of a state is defined to be the integer t (t > 1), such that $p_{ii}^{(n)} = 0$ for all values of n other than t, 2t, 3t, ..., and t is the largest integer with this property. In the two examples, there are no states with a period. In the following Markov chain, the transition diagram of which is presented, all states have period 2.



The proof that the Markov chain contains period 2 follows from the single-step and multi-step transition matrices:

$$P = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = P^{(3)} = P^{(5)} \text{ etc.}$$

$$P^{(2)} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = P^{(4)} = P^{(6)} \text{ etc.}$$

Thus, $p_{ii}^{(n)} = 0$ for n=1, 3, 5 etc. and $p_{ii} > 0$ for n=2, 4, 6 etc.

8.4 Long-run properties of Markov chains

For the first example, the four-step transition matrix was presented earlier. This transition matrix can be used to derive $P^{(8)}$ (= P^4 P^4).

$$P^{(8)} = \begin{bmatrix} 0.263 & 0.211 & 0.526 \\ 0.263 & 0.211 & 0.526 \\ 0.263 & 0.211 & 0.526 \end{bmatrix}$$

Because all three rows are similar, the probability of being in state j after 8 weeks seems to be independent of the initial state vector. If initially all sows are in state 1 (X_0 = {100, 0, 0}), the state vector 8 weeks later will be X_8 = {26.3, 21.1, 52.6}. If the initial state vector X_0 had been {33.3, 33.3, 33.3}, X_8 would also have been {26.3, 21.1, 52.6}.
$P^{(8)}$ can be used to calculate $P^{(16)}$. All entries in $P^{(16)}$ are equal to those in $P^{(8)}$. In case of an irreducible Markov chain, as in the first example, $\lim_{n\to\infty} p_{ij}^{(n)} = \pi_j$ exists and is independent of i. The π_js are called the **steady-state probabilities** of the Markov chain. The term steady-state probability means that the probability of finding the process in state j after a large number of transitions is independent of the initial probability distribution defined over the states and tends to the value π_j. Steady-state probabilities do not imply that the process settles in one state. The process continues to make transitions from state to state (the transition probability from state i to state j is still p_{ij}).
If the distribution over states has reached the steady-state distribution represented by the $__j$s at time n, the distribution over states at time n+1 is the same. This characteristic can be used to derive directly the vector Π containing the stationary probabilities π_j, instead of making all the necessary time steps. The following set of equations needs to be solved: $\Pi = \Pi P$ and $\Sigma_j \pi_j = 1$. For the first example these include:

$\pi_1 = 0.2\,\pi_1 + 0.25\,\pi_2 + 0.3\,\pi_3$
$\pi_2 = 0.8\,\pi_1$
$\pi_3 = 0.75\,\pi_2 + 0.7\,\pi_3$
$1 = \pi_1 + \pi_2 + \pi_3$

Solving the last four equations provides the simultaneous solutions: $\pi_1 = 0.263$, $\pi_2 = 0.211$ and $\pi_3 = 0.526$, which are the results that appeared before in matrix $P^{(8)}$.
When a Markov chain consists of more than one recurrent class of states, the steady-state distribution or limiting distribution over states is no longer independent of the initial state vector X_0. So, $\lim_{n\to\infty} p_{ij}^{(n)} = \pi_{ij}$ and is no longer independent of i. For the second example, the transition matrix with the steady-state probabilities ($P^{(\infty)}$) is as follows:

$$P^{(\infty)} = \begin{bmatrix} 0 & 0 & 0.8 & 0.2 \\ 0 & 0 & 0.8 & 0.2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

If in the initial state vector all animals are susceptible ($X_0 = \{100,0,0\}$), ultimately 80% of the animals will be immune and 20% will have died. When initially 50% of the animals are immune and 50% susceptible, then in the long run 90% of the animals will be immune and 10% will have died owing to the disease. These figures can also be derived from the transition diagram.
If states i and j are recurrent states belonging to different classes, then $p_{ij}^{(n)} = 0$ for all n. In the second example, states 3 and 4 are recurrent states belonging to different classes. Therefore, $p_{34}^{(n)} = 0$ and $p_{43}^{(n)} = 0$, for all n. State j is a transient state, when $\lim_{n\to\infty} p_{ij}^{(n)} = 0$ for all i. In the second example, states 1 and 2 are transient states; $\lim_{n\to\infty} p_{i1}^{(n)} = 0$ and $\lim_{n\to\infty} p_{i2}^{(n)} = 0$, for all i (see matrix above).

Exercise

With the computer case on Markov chains (in Chapter 19) you can practise the principles of Markov chains: setting up a transition matrix and defining whether the different states are recurrent, transient or absorbing You will also see an example of calculating the steady state in one step, as explained in section 8.4. After the introduction, an example with mastitis is worked out. There are different strategies of changing the current mastitis situation in the herd. You have to use the Markov chain approach to determine which strategy is the best. Hereafter, the Markov chain is extended with dynamic transition rates, indicating that the probability of infection is dependent on the number of animals infected in the previous period (this is a more realistic, but also a more complicated way of using Markov chains). The exercise takes approximately 45 minutes.

8.5 Simulation of herd dynamics

A dynamic probabilistic model was developed in order to calculate the effects of different management strategies with respect to production, reproduction and replacement on the technical and economic results of an individual sow herd (Jalvingh, 1993). Central in the model is the simulation of herd dynamics using a modified Markov chain.

8.5.1 Description of the Markov chain model

The sow herd is described in terms of states the animals can be in and the possible transitions between states and the corresponding probabilities. Taking into account model objectives, time interval between transitions is set at 1 week. The states that are included are related to the (re)production cycle of the sows. Table 8.3 presents the state variables that were used to describe the states from weaning to weaning. To cut down the number of states per cycle, the number of state variables used in the second part of the gestation period is reduced. The total number of states from weaning to weaning is 156. Furthermore, the state variable 'cycle number' is used to represent sows of different ages, varying from 1 to 10.

Table 8.3 Possible values of the state variables used to describe states within a cycle of the sow (from weaning to weaning). State variables used are dependent on stage in cycle. Time unit in the model is 1 week.

	State variables[a]					
Stage in cycle	i^b	j	k	l^b	m^b	n
Weaning - insemination	1-3					
Insemination - halfway gestation		0-6		1-4	1-12	
Halfway gestation - farrowing		7-16				0-1
Farrowing - weaning			1-4			

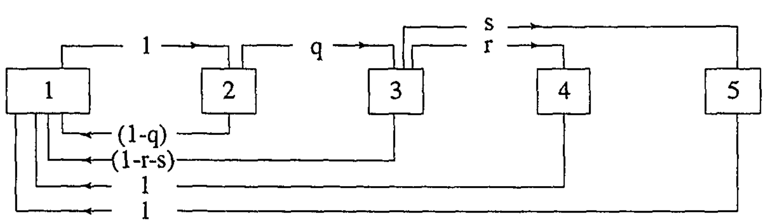
[a] The following state variables are used:
- i: time after weaning;
- j: time after insemination;
- k: time after farrowing;
- l: number of inseminations performed during this cycle;
- m: interval weaning - insemination;
- n: pregnant or not.

[b] Upper limit number of classes depends on user-defined input values; given number is maximum.

By adding extra state variables to the model the production history of the sows can be taken into account. Production level at last farrowing and production level at second last farrowing are included as state variables. So, when taking decisions for individual sows, productivity of the sow can be taken into consideration. Both these extra state variables vary from 4 to 16 pigs born alive. The maximum number of states in the model is 156 x 10 x 13 x 13 = 263 640. In this chapter, the state variables referring to production level will not be used, leaving a model with 156 x 10 = 1560 states.

For each state mentioned above, the probability of making the next transition to each other state has to be specified. Herd dynamics is a result of the interaction between biological variables and management strategies. Therefore, transition probabilities are dependent on input parameters referring to **biological variables** on the one hand (eg, pregnancy, oestrus detection and involuntary culling rates) and **management strategies** on the other (eg, insemination and replacement strategies). Three categories of transition probabilities are considered: (1) reproduction, (2) involuntary disposal, and (3) production level.

At each time step, individual animals are either retained or culled. If a sow is culled, it is replaced by a replacement gilt (6 months old). A replacement gilt can stay in the herd until it is culled and replaced or until it has reached the maximum allowable litter (10 in this case). The following transition diagram is a simplified representation of the Markov chain, which has the same characteristics as the sow herd model. Sows are either retained or culled. If a sow is retained, she has the possibility of going to more than one state (eg, in case of inseminated and not inseminated). State 1 represents the replacement gilt.



The transition diagram shows that all states communicate. Therefore, and when transition probabilities are stationary, the steady-state probabilities are independent of the initial state vector. No matter how the animals were initially distributed over the states, the limiting distribution over states is always the same. The limiting distribution is in fact equal to the distribution of a replacement gilt over all states during her life. The steady-state probabilities are recalculated to represent a herd with a certain size. Due to the ageing of sows, only a few transitions are possible for each state. The transition matrix has a great many rows and columns, containing per row, therefore, only a couple of non-zero entries. The Markov chain has been programmed to allow for these typical characteristics.

To evaluate the consequences of changes in herd dynamics, several technical and economic results are derived from the distribution over states. Some variables are derived directly from the steady-state distribution, such as number of litters per sow per year and percentage of reinseminations. For other variables additional technical and economic variables are needed, as in the case of returns (eg, price of piglets and culled sow) and costs (eg, price of feed and replacement gilt).

8.5.2 Model results

For a basic situation representing typical Dutch herds, the technical and economic results of the corresponding steady-state herd were determined (herd I). Appendix 8.1 presents the major technical and economic input variables. The steady-state herd was also determined if pregnancy rates were at a 20% lower level (herd II), and when oestrus detection rate after first insemination was at a 20% lower level (herd III). The results of the three steady-state herds are presented in Table 8.4.

Table 8.4 Major technical and economical results of different steady-state herds

	Herd		
	I[a]	II[a]	III[a]
Technical results			
Average number of sows	130	130	130
Litters per sow per year	2.32	2.17	2.30
Pigs born alive per litter	10.6	10.5	10.6
Pigs sold per sow per year	21.0	19.5	20.8
Culling rate sows (%)	49.3	69.2	51.7
Reinseminations (%)	11.5	23.7	10.0
Economic results (US$ per sow per year)			
Returns			
- piglets sold	1224	1138	1216
- sows and gilts culled	119	162	125
Costs			
- replacement gilts	152	214	159
- feed sow	328	329	328
- feed piglets	264	246	263
Gross margin	599	511	589
Gross margin herd (US$)	77872	66416	76641

[a] Herd I: basic situation; Herd II: pregnancy rates at 20% lower level; Herd III: oestrus detection after first insemination at 20% lower level.

In the basic situation (herd I), the number of litters per sow per year is 2.32, the number of pigs sold per sow per year is 21.0 and the annual culling rate in sows is 49.3%. Resulting gross margin per sow per year is US$599. If pregnancy rates are at a 20% lower level (herd II), percentages of reinseminations and annual culling rate increase. This results in a reduction in number of litters per sow per year (minus 0.15) and number of pigs sold per sow per year (minus 1.5). The reduction in gross margin is US$88 per sow per year. If oestrus detection rate after first insemination is at a 20% lower level (herd III), fewer sows that have been inseminated but have not become pregnant will be seen in oestrus again. Since oestrus detection rate before first insemination and pregnancy rate are still at a high level, the effects on the results are minimal. The number of litters per sow per year has decreased (minus 0.02). Gross margin per sow per year is US$9 lower than in the basic situation.

The model has been used to compare different insemination strategies (when to stop inseminating if a sow fails to conceive). Besides looking at the herd structure in its stationary state, herd dynamics can be studied over time. The model can be used to study how a herd approaches a new steady state, for instance, when some transition probabilities are modified.

The modelling approach developed for swine was also applied to dairy cattle. Transitions take place at monthly intervals. Month of calving was included as an additional state variable, and, therefore, the period of all states in the Markov chain is 12. The dairy herd model also focuses on the evaluation of different calving patterns, and on the comparison of different strategies to actually change the calving pattern of the herd (Jalvingh, 1993).

8.6 Concluding remarks

In animal health economics, Markov chains are especially used to simulate contagious disease control. The probability of becoming infected is often assumed to depend on the fraction of herds or animals being infected during the previous time period. In that case, the property of stationary transition probabilities does not hold any more. Such a modified version of the Markov chain approach is called the State Transition approach (Dijkhuizen, 1989).

The (modified) Markov chain approach is in fact a stochastic model using probability distributions, taking into account uncertainty about the future behaviour of the system. Another approach to simulate disease spread and herd dynamics is stochastic simulation using random elements (ie, Monte Carlo simulation), as is described in Chapter 9. The Markov chain approach provides the **expected value** of the results by carrying out a single run. In the approach using random elements multiple runs are needed to obtain a reliable estimate of the average results. An advantage of the multiple runs is the information that can be obtained about the **standard deviation** of the results, making statistical tests and non-neutral risk analysis possible. An advantage of the approach using probability distributions instead of random elements is that sensitivity analyses can easily be carried out. The differences between stochastic simulation using probability distributions and using random elements were described extensively in Chapter 5.

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Appendix 8.1

Table A8.1 Basic values of biological input variables that determine transition probabilities concerning reproduction

Variable	Basic value
Distribution of first observable oestrus	
week 1 after weaning	80
week 2 after weaning	20
week 3 after weaning	0
Oestrus detection rate (%)	
before first insemination	98
after first insemination	90
Farrowing rate (within cycle) (%)	
after insemination 1	85
after insemination 2	65
after insemination 3	50
after insemination 4	40
Distribution over 'reasons' for not conceiving	
in oestrus after 3 weeks	90
in oestrus after 6 weeks	0
abortion	7
not pregnant in farrowing house	3

Table A8.2 Cycle-specific input values concerning transition probabilities and technical and economic results

Cycle number	Involuntary disposal (%)	Pigs born alive	Piglet mortality (%)[a]	Live weight sow (kg)
0	10			140
1	2	9.6	13.0	140
2	8	10.3	12.0	160
3	8	10.8	13.0	175
4	7	11.1	13.0	188
5	9	11.2	14.0	196
6	11	11.1	14.0	200
7	13	11.0	14.0	200
8	15	10.9	15.0	200
9	17	10.8	15.0	200
10	18	10.7	15.0	200

[a] Piglet mortality rate before weaning; mortality rate after weaning: 1.5%.

Table A8.3 Economic input variables and their basic values

Variable	Basic value
Feeder pig price (US$/head)	58
Feed price (US$/100 kg)	
gilts and non-lactating sows (EV/kg[a] = 0.97)	25
lactating sows (EV/kg = 1.03)	25
pigs	42
Slaughter value (US$/kg)	
cycle 0	1.40
cycle 1	1.28
cycle 2 and higher	1.22
Price replacement gilts (US$/head)	278

[a] EV/kg = 1 = 8786 kJ net-energy for fat production.