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Soil mixing intensity measurements

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Summary

Soil mixing intensity is strongly related to the lump size (as defined in the text) of the mixing components. A method is proposed for the determination of the mean lump size, based on the frequency distribution of sample composition. This frequency distribution depends on the ratio of sample size to lump size, indicated as α. With the use of a model the theoretically expected probability curves of sample composition are calculated for different values of α. Comparison between the calculated and the experimentally determined distribution leads to the value of mean lump volume and thus to a numerical expression for mixing intensity. The proposed method was tested on different mixing procedures of sand and peat, but can be applied for all mixing systems as long as a physically or chemically distinctive property between the mixing components exists.

Introduction

Mixing of soil is a process occurring continuously in the upper layer of the earth as a result of biological activity and human effort. Although the first mentioned mixing forces are very important with respect to the total volume of soil particles displaced, most attention has been paid to the mixing effect of tillage. This imbalance of interest must certainly be ascribed to the fact that soil mixing as a result of tillage can be influenced relatively easily, e.g. by the choice of construction of the implements or of the soil conditions at the moment of tillage application.

Tillage results are usually studied by transposition measurements of soil particles or soil layers. To this purpose several methods have been suggested. Application of radioisotopes for tracing the soil constituents was described by Njøs and Steenberg (1962). Wiersema (1961) proposed addition of iron filings to the soil and magnetic collection of these filings after tillage. Perdok (1969) studied displacement of soil layers after deep tillage by piling-up sand-filled plastic pipes vertically into the soil, the pieces each having a special code. In this way transposition of soil material both in vertical and horizontal direction could be concluded from the positions of the pieces after recollection.

It is the purpose of this paper to deal with a method for the measurement of soil mixing intensity. Mixing intensity is strongly correlated with the size of the lumps of the different mixing components, the intensity being higher with smaller lump size. A lump then defined as a joint piece of the same component without pollution of other components, and is thus not related to the pieces in which the soil is cut or divided naturally or by implements.

The proposed method offers an opportunity for the determination of the mean lump size (as defined above) on the basis of the cumulative frequency distribution of the composition of samples. As a necessary prerequisite one of the components should be distinguished from the other either physically or chemically by means of naturally prevailing differences in properties (e.g. organic matter content, $CaCO_3$ content, Cu content) or by means of artificial differences as can be imposed by marking with radio-isotopes.

Theoretical considerations

Consider two different soil layers, e.g. pure sand on top of a layer of pure peat, each with a thickness of 40 cm. After mixing these layers by deep soil tillage and sampling of the resulting profile the composition of the samples will depend on the size of the sample in relation to the intensity of the applied mixing procedure as reflected in the lump size. If in the above case very large samples (e.g. of several m^3) would be taken and analyzed all samples would have the same composition namely 50 % (v/v) sand and 50 % (v/v) peat. If the mixed profile would be sampled with a large number of very small samples (e.g. of one cm^3) none or only very few of these samples would contain both sand and peat; thus half of the total number of samples would contain 100 % (v/v) sand, the other half 100 % (v/v) peat. These both samplings would result in the cumulative frequency distributions of sample composition as indicated by Curves 1 and 4 respectively of Fig. 1. It will be clear that intermediate ratios of sample size to lump size will result in, for example, Curves 2 and 3 of Fig. 1. Thus the ratio of the sample size to the lump size is reflected in the course of the cumulative frequency distribution of sample composition. If these distributions are known for different values of the ratio meant, a comparison of the experimentally determined frequency distribution of sample composition in samples of known size with the theoretically predicted distributions, will lead to the value of the lump size and thus to a numerical expression of the mixing intensity.

The probability curves of sample composition can be calculated theoretically with mixture models. For a detailed information on the model calculations, presented here in abbreviated form, the reader is referred to Stol (1971). Fig. 2A represents the cross-

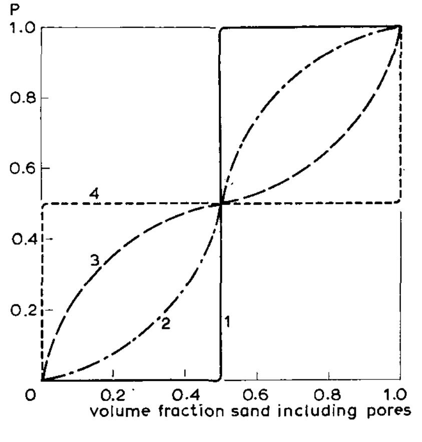

Fig. 1. Probability distributions of sample composition for different ratios of sample size to lump size. Curve 1: sample ≫ lump; Curve 4: sample ≪ lump; Curves 2 and 3: intermediate values of sample to lump ratio

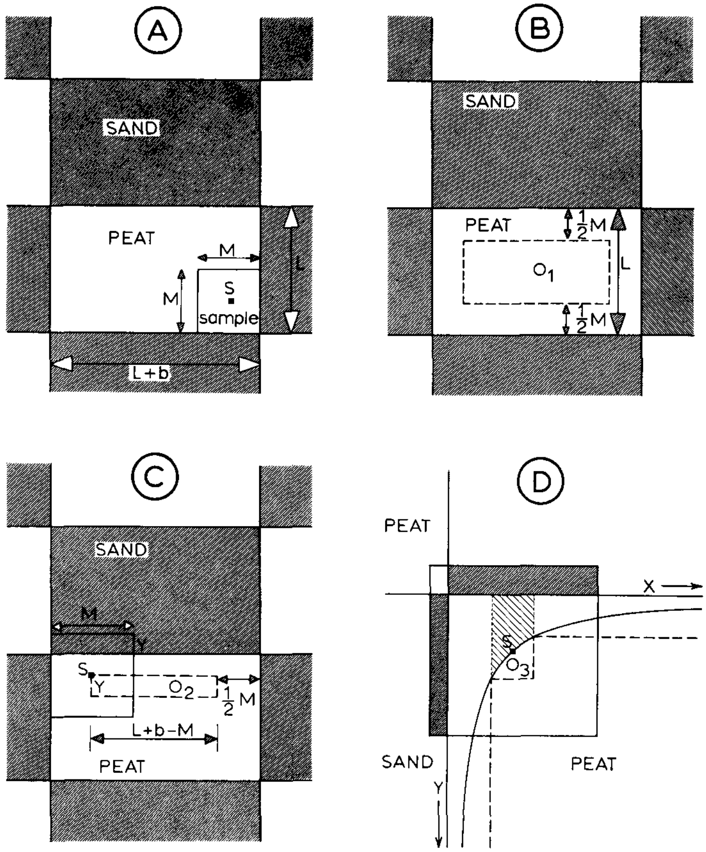

Fig. 2. Several cross-sections of a mixture model used in the calculations

section of such a model in which lumps of equal size of both components occur alternately in all directions. The dimensions of the lumps are L and L + b for the vertical and horizontal direction respectively. The model is assumed to be sampled with a square sample with sides M (cf. Fig. 2A). The ratio between sample size and lump size is indicated as $M/L = \alpha$ for the vertical direction and $M/(L + b) = \beta$ for the horizontal direction. In the case of cubic lumps $\alpha = \beta$ as $b = 0$. Furthermore the composition of the sample is expressed as p, representing the volume fraction of sand. The calculations are performed for a two-dimensional system as given in Fig. 2.

The composition of the sample depends on the position of the sample centre S with respect to the smallest unit of the system, i.e. the cross-sectional area of one peat lump and an adjacent sand lump. If S is an element of area O_1 indicated in Fig. 2B, the composition of the system will always be the same ($p = 0$). It will be clear that pure sand ($p = 1$) in the sample will have the same probability. The probability of such a situation is indicated by the ratio of area O_1 to the cross-sectional area of the system unit:

$$P(\underline{p} = 0) = \frac{(L - M)(L + b - M)}{2L(L + b)} = P(\underline{p} = 1) \tag{1}$$

Substituting the defined values α and β into Eq. 1 gives:

$$P\,(\underline{p} = 0) = {}^1/_2\,(1 - \alpha)\,(1 - \beta) = P\,(\underline{p} = 1) \tag{2}$$

and for a system of cubic lumps:

$$P\,(\underline{p} = 0) = {}^1/_2\,(1 - \alpha)^2 = P\,(\underline{p} = 1) \tag{3}$$

Eq. 3 thus allows calculation of the probabilities indicated, for all different values of α for which the model is valid ($\alpha \leqslant 1$).

All other discrete probabilities equal zero. Consider for instance $P\,(\underline{p} = {}^1/_4)$. The corresponding sample composition is found if S moves to the exterior of the peat lump until one quarter of the sample area is occupied by sand. As is indicated in Fig. 2C for the top of the peat lump, S should be moved over a distance Y in such a way that

$$YM/M^2 = {}^1/_4 = p \tag{4}$$

This condition prevails if S is situated on the upper broken line of Fig. 2C. As the surface area of a line equals zero $P\,(\underline{p} = {}^1/_4) = 0$. Thus for sample compositions $0 < p < 1$ only continuous probabilities can be calculated.

Surface area O_2 of Fig. 2C represents a part of $P\,(0 < \underline{p} \leqslant {}^1/_4)$. In fact $P\,(0 < \underline{p} \leqslant {}^1/_4)$ is found from the surface areas:

1. On the top and lower side of the peat lump a rectangle as indicated by O_2 in Fig. 2C with sides Y and $L + b - M$;
2. On the right and left side of the peat lump a comparable rectangle with sides Y and $L - M$;
3. In the four corners of the peat lump a figure as indicated by O_3 in Fig. 2D.

The contribution of 1 and 2 to $P\,(0 < \underline{p} \leqslant {}^1/_4)$ is easily found as:

$$\frac{2Y\,(L + b - M)}{2L\,(L + b)} + \frac{2Y\,(L - M)}{2L\,(L + b)} \tag{5}$$

This gives after conversion, substitution of $Y = pM$ according to Eq. 4 and substitution of the definitions of α and β

$$p\alpha\,(1 - \beta) + p\beta\,(1 - \alpha) \tag{6}$$

and for cubic lumps

$$2\,p\alpha\,(1 - \alpha) \tag{7}$$

The surface area O_3 can be found after introducing auxiliary axes as has been done in Fig. 2D and expressing the position of S in (x, y) co-ordinates. Applying that on the curved boundary of O_3 the condition $p = {}^1/_4$ prevails, this curved line appears to be a hyperbola of the form

$$xy = {}^1/_4\,M^2\,(1 - 2p) \tag{8}$$

Integration with boundary conditions $x = {}^1/_2\,M(1 - 2p)$ and $x = {}^1/_2M$ leads to the shaded area in Fig. 2D, resulting in the following expression for O_3:

$$O_3 = {}^1/_2\,M^2p + {}^1/_4\,M^2\,(1 - 2p)\ln\,(1 - 2p) \tag{9}$$

In a manner comparable with that in Eq. 5 the contribution of 3 to $P\,(0 < \underline{p} \leqslant {}^1/_4)$ is now found as:

$$\alpha\beta p + {}^1/_2\,\alpha\beta\,(1 - 2p)\ln\,(1 - 2p) \tag{10}$$

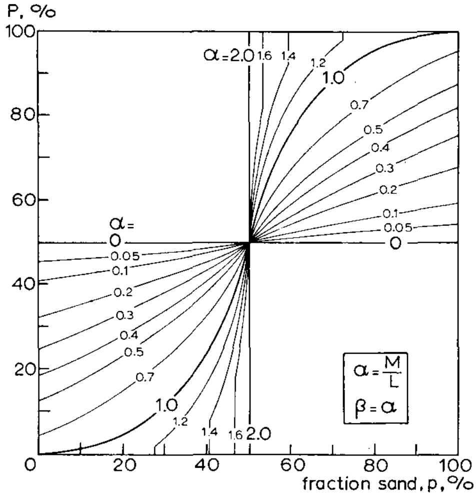

Fig. 3. Calculated probability curves of sample composition for a range of α-values

and for cubic lumps:

$$\alpha^2 p + {}^1/_2\, \alpha^2 (1 - 2p) \ln (1 - 2p) \qquad (11)$$

The probability P is finally found by addition of Eq. 7 and 11:

$$P\,(0 < \underline{p} \leqslant {}^1/_4) = (2\alpha - \alpha^2)p + {}^1/_2\, \alpha^2(1 - 2p) \ln (1 - 2p) \qquad (12)$$

Comparable derivations can be made for p-values between $^1/_2$ and 1. Recapitulation of all equations necessary for the theoretical calculation of the probability curves for cubic lumps and $\alpha \leqslant 1$ gives:

$$P\,(\underline{p} = 0) = {}^1/_2\, (1 - \alpha)^2 \qquad (3)$$

$$P\,(0 < \underline{p} \leqslant p) = (2\alpha - \alpha^2)p + {}^1/_2\, \alpha^2(1 - 2p) \ln (1 - 2p) \qquad (0 < p \leqslant {}^1/_2) \qquad (12)$$

$$P\,({}^1/_2 \leqslant \underline{p} < p) = (\alpha - {}^1/_2\, \alpha^2)\,(2p - 1) - {}^1/_2\, \alpha^2(2p - 1) \ln (2p - 1) \qquad ({}^1/_2 \leqslant p < 1) \qquad (13)$$

$$P\,(\underline{p} = 1) = {}^1/_2\, (1 - \alpha)^2 \qquad (3)$$

Applying these equations for a number of α-values the probability curves as presented in Fig. 3 are found.

Also for $\alpha > 1$ (sample size larger than lump size) model calculations can be performed showing that distribution 1 of Fig. 1 must be expected at each even integer number for α.

Experimental results

A soil consisting of 40 cm almost pure sand on top of peat was mixed to a depth of 80 cm by two different soil improvement tools namely a mixing rooter and a rotary mixer (for this experiment cf. Beuving and de Haan, 1968). As a result of their different construction the machines achieve a different mixing intensity. After treatment with the mixing rooter the original soil materials are maintained in relatively large

Depth cm	0		10		20		30		40		50		60		70		80		90		100		110	120 cm
0 - 5	33.2		37.8		51.0		63.4		51.7		31.5		31.6		30.4		25.7		34.0		19.2		27.9	
5 - 10		50.6		42.2		41.0		39.4		34.7		60.5		30.4		34.7		22.2		17.2		17.5		17.4
10 - 15	1.0		3.5		58.0		100.0		93.3		8.5		1.4		4.3		4.4		2.5		3.7		0.9	
15 - 20		9.4		4.0		75.9		100.0		89.6		14.2		9.2		3.5		2.6		2.7		3.7		13.1
20 - 25	76.7		10.0		4.9		89.3		95.9		100.0		18.4		32.8		4.7		3.1		2.7		16.9	
25 - 30		72.7		45.0		28.5		100.0		100.0		100.0		94.0		65.4		55.0		9.3		52.5		96.7
30 - 35	67.0		17.7		0.3		69.7		89.3		97.1		100.0		80.9		96.7		78.6		43.6		100.0	
35 - 40		60.0		0.3		8.0		100.0		89.4		100.0		77.6		88.9		80.1		83.6		90.5		100.0
40 - 45	41.3		0.1		0.0		90.7		100.0		100.0		100.0		100.0		100.0		54.8		53.0		97.0	
45 - 50		4.0		0.2		70.8		100.0		100.0		100.0		100.0		100.0		69.9		75.8		74.9		82.0
50 - 55	37.5		0.3		21.3		86.3		89.2		100.0		100.0		100.0		100.0		72.0		54.9		97.4	
55 - 60		54.8		0.1		36.3		100.0		84.6		80.4		100.0		84.5		52.4		76.6		3.4		60.1
60 - 65	1.0		2.5		0.0		100.0		88.3		64.7*		81.7		61.5		62.6		80.0		1.2		86.5	
65 - 70		0.8		0.0		24.5		100.0		91.6		28.9		81.9		63.6		81.0		30.3		36.2		22.3
70 - 75	0.0		1.8		9.2		57.6		56.8		0.4		0.0		76.2		87.6		45.2		0.7		0.4	
75 - 80		1.3		9.8		9.7		100.0		48.1		0.0		3.0		65.3		27.6		0.5		1.3		0.8

Table 1. Volumetric sand content (%) of samples of mixing rooter plot.

Depth cm	0		10		20		30		40		50		60		70		80		90		100		110	120 cm
0 – 5	45.2		35.9		54.5		41.2		54.6		52.4		54.1		54.6		48.3		46.8		38.2		34.0	
5 – 10		35.1		34.3		64.8		58.1		24.5		59.3		38.0		48.3		55.0		54.9		48.9		39.3
10 – 15	67.9		54.7		44.7		58.3		36.4		54.2		81.3		59.9		82.0		53.2		62.7		33.9	
15 – 20		70.7		72.1		70.1		57.7		44.6		68.5		70.0		58.9		38.3		54.9		35.1		16.8
20 – 25	70.7		75.2		51.2		54.4		75.8		49.4		53.1		68.7		52.5		43.2		20.7		52.3	
25 – 30		54.2		62.4		71.0		52.4		53.1		44.8		64.9		65.2		57.6		59.8		67.4		32.6
30 – 35	37.4		72.3		63.6		54.2		68.2		45.0		75.8		86.1		90.6		54.4		50.0		58.9	
35 – 40		68.8		62.5		64.8		49.9		59.5		85.2		90.1		62.4		31.8		33.6		64.2		69.7
40 – 45	50.3		92.3		40.6		11.0		49.5		49.3		64.1		76.9		58.1		58.8		57.8		45.9	
45 – 50		2.8		64.8		50.0		14.9		77.5		67.2		85.9		76.0		64.6		47.1		49.2		51.5
50 – 55	27.4		41.4		44.8		7.5		48.3		67.6		19.6		44.7		61.6		6.6		41.3		83.2	
55 – 60		41.3		70.4		49.8		46.4		59.6		45.6		86.8		63.2		62.5		19.7		31.5		54.7
60 – 65	33.9		29.2		48.2		75.7		32.8		34.3		41.9		58.7		49.0		23.3		31.1		32.1	
65 – 70		17.2		45.9		13.3		9.7		22.9		45.4		95.0		59.0		46.2		37.6		21.0		65.6
70 – 75	4.4		0.3		57.9		26.2		13.8		31.0		95.6		58.2		19.4		37.8		22.6		28.0	
75 – 80		4.9		32.4		3.5		32.8		26.9		80.8		40.8		71.0		25.5		17.3		59.9		60.0

Table 2. Volumetric sand content (%) of samples of rotary mixer plot.

lumps as compared to the treatment with the rotary mixer. A description of both tools is given by Wind (1969).

The difference in treatment turned out to have a remarkable influence on the drainage properties of the new profiles. This was indicated by the course of the groundwater level which was always considerably higher in the rotary mixer plot than in the mixing rooter plot at comparable conditions for drain distance and drainage depth. This must be attributed to differences in water storage properties and in saturated water conductivity. The last mentioned values turned out to be about ten times higher for the mixing rooter plot than for the rotary mixer plot (2 m/day against 0.2 m/day; cf. Beuving, 1971). Since both profiles contained equal amounts of the original soil materials at the same density, the differences mentioned must be attributed to the mixing intensity.

Both profiles were sampled intensively by taking 192 samples from a vertical cross-section of 120 × 80 cm. According to the model calculations one should determine the relative amount of sand and peat in samples of infinite thinness. To meet this condition as much as possible, sample dimensions were chosen in such a way that the content of the sample was collected from a relatively large vertical cross-sectional area in comparison to horizontal sampling depth (20 cm² × 2.5 cm).

All samples were analyzed on dry bulk density and on organic matter content, the last being the distinguishing property of both mixing components in this case. This, however, results in data on weight basis whereas volume fractions of the components are required. After plotting for all sample analyses the value of dry bulk density against organic matter content, as was suggested by Schothorst (1968), mean values of the dry bulk density of pure sand and pure peat could be derived. These were used for calculation of the distribution of sand and peat in each sample on volume basis. A correction was made for the impurity of the sand, as this contained in the original profile a very small amount of organic matter (de Haan, 1971).

Tables 1 and 2 represent the experimental results expressed as volume fraction of sand (p) in %, for the mixing rooter profile and the rotary mixer profile respectively. It is seen in Table 1 that after applying the mixing rooter relatively large lumps of the original materials are maintained, as the occurrence of a number of adjacent samples containing either pure sand or pure peat is a common feature. This is in strong contrast with the rotary mixer result (Table 2) where sand and peat are much more equally

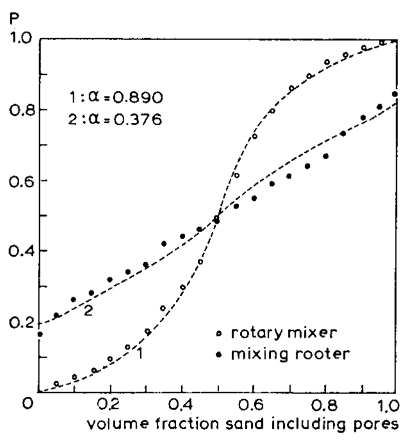

Fig. 4. Experimentally determined frequency distributions for the two mixing procedures applied (points) and calculated probability curves for $\alpha = 0.890$ and $\alpha = 0.376$ (curves).

distributed over the entire profile. Taking the mean value of all data of Tables 1 and 2 one finds 49.7 % and 49.8 %, respectively. Both values are in very good agreement with each other and meet very closely 50 %, expected on the basis of the amounts of both materials in the original profile.

After division of the sand content values into classes and taking the number of samples belonging to a certain class, the cumulative frequency distribution of sample composition is easily found. The results for both plots are represented by the points of Fig. 4. In this figure also two probability curves calculated on model considerations are presented, namely those which fit the experimental results best. As is indicated in Fig. 4 the results of the rotary mixer plot follow extremely well the calculated curve corresponding to an α-value of 0.890. Although this is somewhat less the case with the mixing rooter data and the theoretical curve for α equal to 0.376, also this agreement is still reasonably good.

The value for M being 4.45 cm one thus may conclude that the corresponding values of L for the lumps were:

rotary mixer plot 4.45/0.890 = 5.0 cm and
mixing rooter plot 4.45/0.376 = 11.8 cm

Translating this for cubic lumps one finally finds mean lump volume values of 125 cm^3 and 1643 cm^3, respectively.

Again using the mean lump volume as a measure for comparison of mixing intensity the above result leads to the conclusion that in the situation studied, the mixing intensity of the rotary mixer has been roughly 13 times as high as that of the mixing rooter.

Conclusion

It is realized that in the model considerations several approximations were introduced in comparison to the actual situation in the field. The mixture was assumed to be completely symmetric and calculations were performed for two-dimensional systems only. Although these conditions are certainly not met in practice the method gives a means to express mathematically differences in mixing intensity and thus may be a contribution to study mixing processes.

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