Neth. J. Agric. Sci., 17 (1969) : 71—79

Measuring thermal conductivity of soils under laboratory conditions

A. I. Golovanov

Moscow Hydromeliorative Institute, Moscow, USSR

Received 29 October, 1968

Summary

A series of experiments concerning the determination of the thermal conductivity of soils has been described. When a thin needle used as heating element is placed along the main axis of a cylindrical sample, the thermal conductivity can be computed fairly accurately from a simplified equation (3) when the conditions as given in Fig. 1 are satisfied. For measurements during a period of 100 seconds the diameter of the sample must be at least 4 cm.
If heating elements are placed horizontally, for instance to measure simultaneously the thermal conductivity of different soil layers, it was found that their distance must be at least 10 cm. Concerning the convection of heat it was found that the measurements should preferably not exceed 100 seconds. Finally it was investigated to what extent thermal conductivity measurement can be used to determine flow velocities of water in soils. For coarse sand the method is able to detect real flow velocities higher than some 0.35 cm/min.

Introduction

The thermal conductivity is an important factor in the heat regime of soils and needs to be known for various types of calculations of the heat balance of the earth's surface (Chudnovsky, 1954; Van Duin, 1956; Van Wijk, 1963). Since its magnitude largely depends on the moisture content of the soil it is also used as an indirect method for the determination of the moisture content. Examples of the latter use are given by Van Duin and De Vries (1954), and Youngs (1956). When determining the moisture content of the soil by measuring the thermal conductivity it is important to know the effect of the duration of the measurement, the size of the sample and the amount of energy to be applied. In an experimental setup to measure the flow of water in unsaturated soils, the moisture determination by means of heat conductivity was chosen by the Division of Hydrology of the Institute for Land and Water Management Research at Wageningen, The Netherlands [1]. In order to get some insight in the effect of the above mentioned factors, a series of experiments was carried out. The results are described in this article.

[1] The author spent in 1968 a 9-month period as visiting-scientist at that Institute.

Measuring techniques

One of the methods for the measurement of the thermal conductivity of a soil is based on observing the rise in temperature of an electrically heated cylindrical element (needle) stuck into the soil. For a thin and infinitely long element the temperature field in an infinite medium can be described by the equation

$$\vartheta - \vartheta_0 = -\frac{q}{4\,\pi\,\lambda}\ E_i\left(-\frac{r^2}{4\,\varkappa t}\right) \tag{1}$$

where ϑ = temperature in any arbitrary point at a distance r from the centre of needle in °C;
t = time in sec.;
ϑ_0 = initial ($t = 0$) temperature in °C;
q = steady input of heat per unit length of needle in cal/sec. cm;
λ = thermal conductivity in cal/cm. sec. °C;
$\varkappa$ = thermal diffusivity defined as $\varkappa = \frac{\lambda}{\varrho\, c}$ in cm^2/sec.;
ϱ = density of the soil in g/cm^3;
c = specific heat in cal/g. °C.

For $t > r^2/4\varkappa$ it is possible to simplify equation (1) as

$$\vartheta - \vartheta_0 = \frac{q}{4\,\pi\,\lambda}\left(\ln \frac{4\varkappa}{r^2}\, t - 0.5772\right) \tag{2}$$

If the rise of the temperature $\triangle\,\vartheta$ during time $\triangle\, t = t_2 - t_1$ is known one can calculate the thermal conductivity from the equation

$$\lambda = \frac{q}{4\pi\,\triangle\,\vartheta}\ \ln t_2/t_1 \tag{3}$$

There are several publications in which this method and the apparatuses used have been given (for example De Vries, 1952; Van Drunen, 1949; Janse and Borel, 1965).

Size of soil samples

Under laboratory conditions one deals with soil samples of finite size. The problem is then for what size of samples the simplified equation (3) is valid. It is clear that a small sample kept at a constant temperature at its surfaces will show a smaller rise of temperature inside the sample than an infinite sample. If λ is then calculated by equation (3) the value found will be too high. On the other hand, if the surfaces of the sample are perfectly insulated this will result in a λ too low. Between these two extremes are cases with a certain heat flux through the surfaces, giving smaller errors.

The magnitude of the error introduced by the size of the sample also depends on the duration of the measurement and on the heat properties of the soil. As far as these factors are concerned the error can be estimated theoretically. To this purpose two theoretical solutions can be applied that originally were derived for groundwater flow (see Muskat, 1946). The first one can (after an appropriate conversion) describe the temperature ϑ^* in an infinitely long cylinder which has in its centre a thin linear heat source and a constant temperature ϑ_0 at its surfaces.

$$\vartheta' = 2 \ln \frac{R}{r} - 4 \sum_{n=1}^{\infty} \frac{J_0(\zeta \mu_n)}{\mu_n^2 J_1^2(\mu_n)} e^{-\mu_n^2 \chi/R^2} \tag{4}$$

where $\vartheta' = \frac{4 \pi \lambda}{q} (\vartheta^* - \vartheta_0)$ relative temperature rise;

R = radius of the cylinder in cm;
ζ = r/R;
r = the distance of any arbitrary point from the axis of the cylinder in cm;
χ = $\varkappa t$ in cm^2;
μ_n = roots of the function $J_0(\mu_n) = 0$.

A second solution pertains to the temperature ϑ^{**} of an infinitely long cylindrical sample with insulated surfaces. This solution is:

$$\vartheta'' = 2 \ln R/r + \zeta^2 + \frac{4\chi}{R^2} - 3/2 - 4 \sum_{n=1}^{n=\infty} \frac{J_0(\zeta \mu_n)}{\mu_n^2 J_0^2(\mu_n)} e^{-\mu_n^2 \chi/R^2} \tag{5}$$

where $\vartheta'' = \frac{4 \pi \lambda}{q} (\vartheta^{**} - \vartheta_0)$ relative temperature rise;

μ_n = roots of the function $J_1(\mu_n) = 0$.

A digital computer program was developed to calculate the relative temperature rise for sample radii of R = 1, 2, 3, 5, 7, 10 cm and for a 0.05 cm radius of the needles. The values obtained were compared with the results of equation (1), for which the value $R = \infty$ was assumed (Table 1).

Using these results it was found that the radius R of the samples for which equation (3) can be used for determining λ with an error $|\triangle| < 1\%$, can be represented by the region below the curve in Fig. 1. If measurements are carried out during a period t = 100 sec and $\varkappa = (3 \text{ to } 6). 10^{-3}$ cm^2/sec the radius of the sample must be $R \geqslant 1.1$ to 1.4 cm. For a measuring time t = 300 sec one finds $R \geqslant 1.8$ to 2.5 cm accordingly.

To check these calculations some laboratory experiments have been carried out. The apparatus and techniques used were the same as those described by Janse and Borel (1965). The increase in temperature was taken by a recording millivoltmeter type Servogor. Very fine sand ('Blokzijl' sand) was used, having a dry bulk dentity of 1.16 g/cm³ and a moisture content of 19.4% by volume. Temperature measurements were carried out in copper cylinders R = 1.0, 2.1, 3.0, 5.0, 7.0 and 10.0 cm diameter and a length of 10 cm. The length of the needles was 8.5 cm. Top and bottom of cylinders were carefully insulated by a sheet of poly-urethane foam to obtain an exact radial flow of heat. In one series of measurements the walls of the cylinders were kept at a constant temperature by placing them in a water-thermostat chamber restricting temperature fluctuations to ± 0.01 °C. In another series the walls of cylinders were insulated by a glass-wool blanket. The results of these measurements

Table 1 Relative temperature rise according eq. (1), R = ∞ ; eq. (4), ϑ′ ; and eq. (5), ϑ″

χ cm^2	R = 1 cm		R = 2 cm		R = 3 cm		R = 5 cm		R = 7 cm		R = 10 cm		R = ∞
	ϑ'	ϑ''	ϑ'	ϑ''	ϑ'	ϑ''	ϑ'	ϑ''	ϑ'	ϑ''	ϑ'	ϑ''	$(\vartheta-\vartheta_0)\,\frac{4\pi\lambda}{q}$
0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.03	3.315	3.315	3.315	3.315	3.315	3.315	3.315	3.315	3.315	3.315	3.315	3.315	3.315
0.06	3.998	3.998	3.998	3.998	3.998	3.998	3.998	3.998	3.998	3.998	3.998	3.998	3.998
0.09	4.400	4.400	4.400	4.400	4.400	4.400	4.400	4.400	4.400	4.400	4.400	4.400	4.400
0.12	4.685	4.686	4.685	4.685	4.685	4.685	4.685	4.685	4.685	4.685	4.685	4.685	4.685
0.15	4.906	4.909	4.908	4.908	4.908	4.908	4.908	4.908	4.908	4.908	4.908	4.908	4.908
0.18	5.084	5.095	5.089	5.089	5.089	5.089	5.089	5.089	5.089	5.089	5.089	5.089	5.089
0.24	5.353	5.405	5.376	5.376	5.376	5.376	5.376	5.376	5.376	5.376	5.376	5.376	5.376
0.30	5.540	5.674	5.599	5.599	5.599	5.599	5.599	5.599	5.599	5.599	5.599	5.599	5.599
0.45	5.802	6.292	6.003	6.004	6.003	6.003	6.003	6.003	6.003	6.003	6.003	6.003	6.003
0.60	5.912	6.894	6.289	6.293	6.291	6.291	6.291	6.291	6.291	6.291	6.291	6.291	6.291
0.90	5.997	8.094	6.679	6.717	6.696	6.696	6.696	6.696	6.696	6.696	6.696	6.696	6.696
1.20	5.989	9.294	6.925	7.058	6.983	6.984	6.983	6.983	6.983	6.983	6.983	6.983	6.983
1.50	5.991	10.494	7.085	7.372	7.203	7.210	7.206	7.206	7.206	7.206	7.206	7.206	7.206
1.80	5.991	11.694	7.188	7.676	7.379	7.400	7.389	7.389	7.389	7.389	7.389	7.389	7.389
2.40	5.991	14.094	7.298	8.278	7.640	7.722	7.676	7.676	7.676	7.676	7.676	7.676	7.676
3.00	5.991	16.494	7.344	8.878	7.815	8.010	7.899	7.900	7.899	7.899	7.899	7.899	7.899
4.50	5.991	22.494	7.374	10.378	8.046	8.688	8.300	8.311	8.305	8.305	8.305	8.305	8.305
6.00	5.991	28.494	7.377	11.878	8.134	9.356	8.569	8.621	8.592	8.593	8.592	8.592	8.592

are shown in Fig. 2. In this figure the relation between λ_R and λ_∞ is given. The value of λ_R is that computed by equation (3) from measurements in cylinders with radius R respectively 1, 2, 3, 5 and 7 cm, and λ_∞ is that computed from the measurements in the cylinders with radius R = 10 cm. The values of λ were computed for various time intervals namely for t = 10 to 100 sec, for t = 100 to 300 sec and for t = 300 to 1000 sec. Comparison of these data with the calculations by equations (1), (4) and (5) (with $\lambda = 2.2 \,.\, 10^{-3}$ cal/cm. sec. °C; $\varkappa = 4.87 \,.\, 10^{-3}$ cm²/sec) confirmed our theoretical conclusion about influence of the size of soil samples.

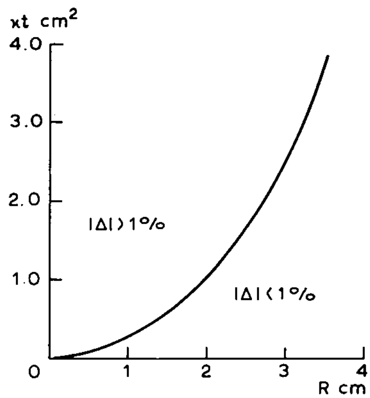

Fig. 1 Minimum radius of cylindrical soil samples in which the thermal conductivity λ can be determined by eq. (3) with an error $|\triangle| < 1\,\%$

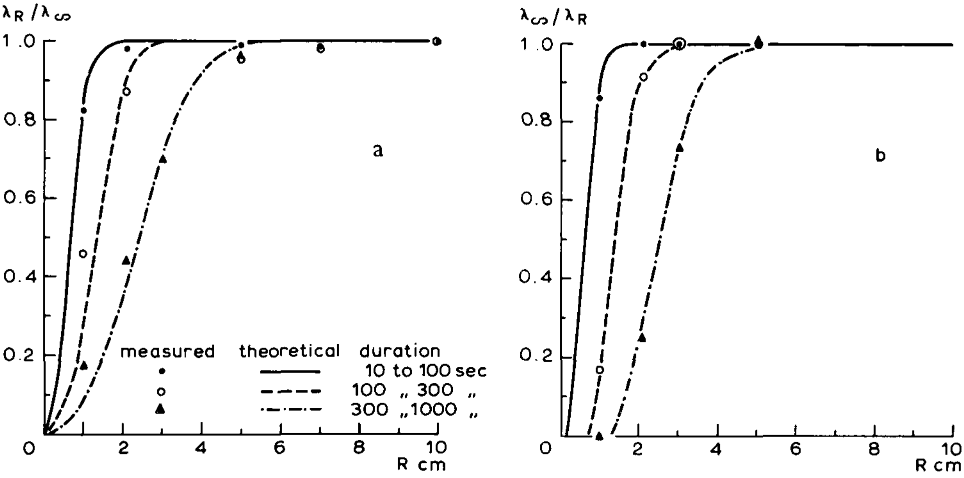

Fig. 2 Influence of the radius of the sample R on the determination by means of eq. (3) of the thermal conductivity of a soil. a: samples with insulated walls; b: samples with the walls at a constant temperature

Sometimes measurements of thermal conductivity have to be carried out in cylindrical samples longer than the length of the needle, for instance if one wants to know heat properties of the soil at various depths. In these cases the needle is to be inserted perpendicular to the main axis of the cylinder and in that case one deals with a rather complicated three-dimensional flow of heat, for which no theoretical solutions are available. In order to check in how far equation (3) may be used for the determination of thermal conductivity under such conditions, a series of measurements were done in cylinders with a radius of 6 cm and of 10 cm length. The 'Blokzijl' sand in these experiments had a dry bulk density of 1.38 g/cm^3 and moisture contents of respectively 11.6% and 35.6% by volume. Comparison of values of λ taken from these experiments with those computed from measurements with the needle through the main axis of the cylinders did show, that during 100 sec observations the difference between the two values was not larger than 2%. For example in sand with a moisture content of 11.6%, 15 observations obtained from measurements with the needle through the main axis of the sample gave a mean value of $\lambda = 2.338 \,.\, 10^{-3}$ cal/cm. sec. °C with a standard deviation $\sigma = 0.021 \,.\, 10^{-3}$ cal/cm. sec. °C. A series of 12 observations with the heating needle perpendicular to the main axis of the sample gave a mean value of $\lambda = 2.435 \,.\, 10^{-3}$ with $\sigma = 0.011 \,.\, 10^{-3}$. A statistical analysis gave as a result $\triangle \lambda = 0.047$ with a standard deviation $\sigma_{\triangle} = 0.024$, so no significant difference between the results of the two ways of measuring was found.

When the needle is inserted perpendicular to the main axis, it is also of interest to know the influence of the length of the sample on λ as determined by means of equation (3). For this purpose cylinders were used with a radius of 6 cm and length of respectively 5, 7, 10, 15 and 20 cm. The surface of the samples were carefully insulated. In Table 2 mean values from 6 repetitions of λ are given. The data in Table 2 show, that in the case of measurements with some horizontally inserted needles in a soil column, the vertical distance between the elements must for practical

Table 2 Influence of the length of cylindrical samples of a sandy soil on the determination of the thermal conductivity according eq. (3), the heating needle inserted perpendicular to the main axis

Length of cylinder (cm)	Moisture content 11.6 % by vol.		Moisture content 35.6 % by vol.	
	$\lambda \cdot 10^3$	λ_l/λ_{10}	$\lambda \cdot 10^3$	λ_l/λ_{10}
5	2.25	0.926	3.13	0.954
7	2.36	0.971	3.19	0.972
10	2.43	1.000	3.28	1.000
15	2.43	1.000	—	—
20	2.46	1.012	—	—

purposes be 10 cm or more in order to prevent mutual influences. This conclusion holds only if equation (3) is used for the calculation of λ, and the duration of measuring is 100 sec.

Influence of convection on heat transport

If some part of a porous medium containing water is heated, there will be a supplementary heat transport due to the convection flow of the water. The effect of this phenomenon depends on the duration of heating and on the moisture content of the porous medium. To get an insight in this effect some measurements have been carried out with 'Blokzijl' sand, having a dry bulk density of 1.3 g/cm^3 and moisture contents of respectively 0.3, 13.9, 26.2 and 36.3 % by volume. Samples were prepared in cylinders with a length of 10 cm and a radius of 10 cm. The ends of the samples were insulated. For heating, currents of I = 0.036 to 0.800 amp. were applied. The temperature rise after 100 sec. was $\triangle \vartheta$ = 0.35 to 52.0 °C respectively and after 300 sec. $\triangle \vartheta$ = 0.39 to 58.0 °C. In Fig. 3 the relation between λ and the current is shown. The values of λ are those determined by observations of temperature rise during 10 to 100 sec. Scatter values of λ from its mean show that there is practically no dependency of λ on I. This means, that the influence of convection on heat transport during this time is negligeable. In almost all experiments, however, a relatively smaller rises in temperature occurred from 100 to 300 seconds than in the period from 10 to 100 seconds. This implies an increasing λ and hence an increasing heat transport in the first mentioned period. This is shown in Fig. 4 in which the ratio $\lambda_{300}/\lambda_{100}$ is given for different currents and different moisture contents. The lines in this figure, drawn by the method of the least squares, are sloping into the direction of an increasing current. This increase of λ is proportional with the moisture content of the soil and it is therefore reasonable to ascribe this to supplementary heat transport caused by a convection flow of the water. For very dry sand with a moisture content of 0.3 % by volume it may be due to convection of water vapour and air. Fig. 4 does show a decreasing influence of convection at higher currents, that is at higher temperatures (at current of 0.5 to 0.8 amp. there is practically no increase of λ). This phenomenon can be explained by a decreasing of λ due to dessication of the soil near the needle.
From the above it can be concluded that over short time intervals (near 100 sec.) should be used to determine the thermal conductivity of the soil.

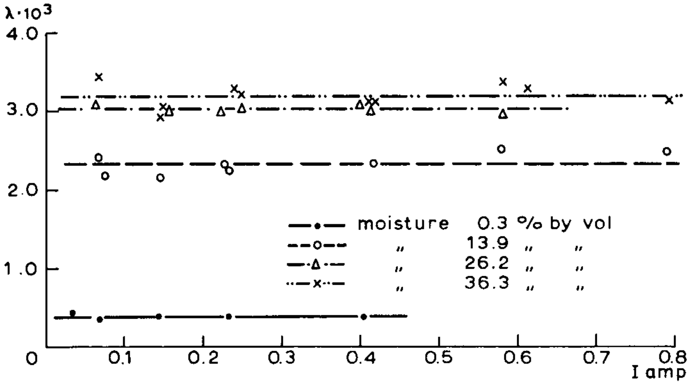

Fig. 3 Relation between thermal conductivity determined during 10 to 100 seconds and current I

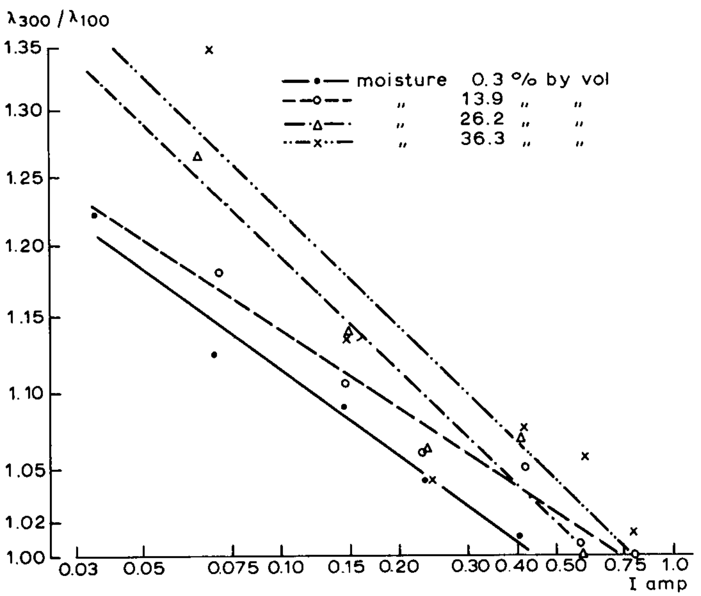

Fig. 4 Relative increase in the thermal conductivity due to convection

Influence of water flow on heat transport

Finally the influence of water moving through porous media on the determination of their thermal conductivity was considered. To this end the thermal conductivity was measured in a cylindrical column with a length of 40 cm and a radius of 6 cm filled with coarse sand (fraction 0.60 to 0.85 mm, hydraulic conductivity 59 meters/day) in which different flow velocities of water were created under saturated conditions. The results of the measurements are shown in Fig. 5. In this figure λ_v is the

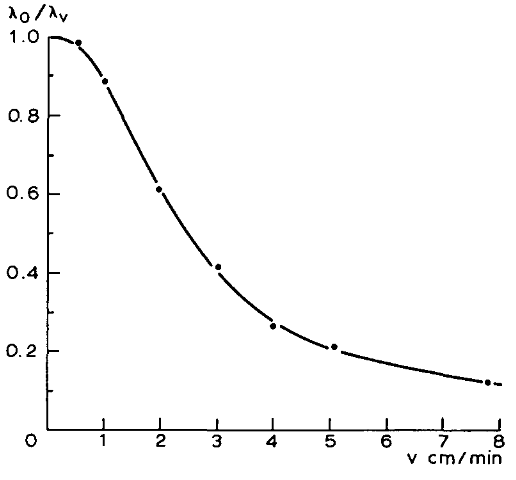

Fig. 5 Relative increase in thermal conductivity of coarse sand resulting from water flow through the sample

thermal conductivity of the medium, with water flowing through it at a velocity v, λ_o is the thermal conductivity with that velocity equal to zero. The flow velocity was defined as the mean real flow velocity in the sample measured at a temperature of 10 °C and it was calculated by the formula

$$v = \frac{Q}{Fp}$$

where Q = discharge of water through the column in cm^3/min;
F = cross-sectional area of the column in cm^2;
p = porosity of the sand in %.

Each of the points on Fig. 5 presents a mean of 4 measurements. In all the measurements an increase of λ due to additional heat transport by the moving water could be observed. A noticeable increase (some 1.5 to 2%) of λ starts already when the flow velocity is 0.35 cm/min. The relation between λ_o/λ_v and v can be used, for example, to registrate the velocity of moving water through porous media, provided that the real flow velocity is larger than about 0.35 cm/min.

Acknowledgement

The author is greatly indebted to Dr. J. Wesseling and Mr. K. E. Wit of the Division of Hydrology, and Mr. W. van Doorne of the Section Mathematics for their aid and advice throughout his work at the Institute for Land and Water Management Research at Wageningen, The Netherlands.

References

Chudnovsky, A. F., 1954. Teploobmen y dispersnykh sredakh (Heat transfer in despersed media). Gostekhizdat. Moscow.

Drunen, F. C. van, 1949. Meting van de warmtegeleiding in vloeistoffen (Measuring thermal conductivity in liquids). Thesis, Utrecht.
Duin, R. H. A. van, 1956. Over de invloed van grondbewerking op het transport van warmte, lucht en water in de grond (On the influence of tillage on conduction of heat, diffusion of air and infiltration of water in soil). Thesis, Wageningen. *Versl. Landbk. Onderz.* 62. 7. Pudoc, Wageningen.
Duin, R. H. A. van & Vries, D. A. de, 1954. A recording apparatus for measuring thermal conductivity and some results obtained with it in soil. *Neth. J. Agric. Sci.*, 2: 168–175.
Janse, A. R. P. & Borel, G., 1965. Measurement of thermal conductivity in situ in mixed materials, e.g. soils. *Neth. J. Agric. Sci.*, 13: 57–62.
Muskat, M., 1946. The flow of homogeneous fluids through porous media. Ann. Arbor. Michigan.
Vries, D. A. de, 1952. Het warmtegeleidingsvermogen van grond (The thermal conductivity of soils). *Meded. Landbouwhogeschool Wageningen*, Netherlands 52. 1.
Wijk, W. R. van, 1963. Physics of plant environment. North-Holland Publishing Company, Amsterdam.
Youngs, E. G., 1956. A laboratory method of following moisture content changes. Rapp. VIe Congres Intern. Sci. du Sol, Paris: 89–93.

REVIEWS

633.2:631.559

J. P. van den Bergh : An analysis of yields of grasses in mixed and pure stands. Thesis, Agric. Univ., Wageningen, 1968. Also published as Agric. Res. Rept No. 714. Pudoc, Wageningen. 71 pp., 50 Figs, 13 Tables, with Dutch summary, price: Dfl. 8.35.

To describe competition species must be compared with a standard. By conventional standards, such as grammes dry matter, tiller number and sq. cm leaf area, two species and even one species cannot be compared in the course of the season. Therefore a standard without dimensions was needed. Dividing yield of a species (expressed in any unit) in mixed culture by that in monoculture gave a figure without dimensions, called relative yield.