Some solutions of the steady state diffusion of carbon dioxide through soils

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Summary

Solutions have been given for the steady state diffusion of carbon dioxide through soils. The general diffusion equation (2) has been solved under the assumption that the relation between the ratio D/D_a (= ratio coefficient of diffusion in soil and in air) and the fraction of airfilled pores is given by (4). The course of the CO_2-production α in the soil profile is given by (5).
In the case of sprinkler irrigation, the moisture content throughout the profile is assumed to be constant (field capacity). The solution is given by (6).
If no sprinkler irrigation is applied, the solutions are given for the decrease of the air-content with the first and second power of the depth (eq. 9). For the first case the solution is given by eq. 20 and *fig.* 3, for the second case the solution is given by eq. 21 and *fig.* 4. The last two figures are based on a total CO_2-production B = 154 mg m^{-2} hr^{-1}.

1. Introduction

Air can be supplied to the soil either by hydrodynamical flow owing to differences in pressure or by diffusion owing to differences in concentration gradients. BUCKINGHAM (1904) pointed out that the hydrodynamical flow is of little importance for the aeration of the soil and that air is supplied mainly by diffusion. Various other investigators (VAN BAVEL, 1951, 1952), BLAKE and PAGE (1948), PENMAN (1940), ROMELL (1922), TAYLOR (1949) arrived at the same conclusion.
Diffusion of gas through a porous medium takes place according to the general equation

$$\frac{\partial q}{\partial t} = \frac{\partial}{\partial z}\left(D\frac{\partial c}{\partial z}\right) + \alpha \qquad (1)$$

where

q = amount of diffusion in gmol cm^{-2} sec^{-1}
t = time in seconds
z = distance in cm
D = coefficient of diffusion in cm^2 sec^{-1}
c = concentration of gas in gmol cm^{-3}
α = amount of gas (gmol) produced or adsorbed per unit time and unit volume. It is taken negative in the case of adsorption and positive in the case of production.

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Since the changes in saturation of the soil are generally slow, as compared with the time in which the air in the upper layer of the soil is changed by diffusion, the aeration of the soil may be described by the equation for the steady state diffusion, derived from eq. 1:

$$\frac{\partial}{\partial z}\left(D\frac{\partial c}{\partial z}\right) + a = 0 \tag{2}$$

For the problem of aeration of the soil this equation is for homogeneous profiles subject to the boundary conditions:

$$\begin{aligned} c &= C_0 \qquad & z &= 0 \\ \frac{\partial c}{\partial z} &= 0 \qquad & z &= L \end{aligned} \tag{3}$$

C_0 being the concentration of the gas in the free atmosphere.

Solutions of the equation are mainly dependent on a and D which are, in general, decreasing with depth. In the next sections some solutions will be given in which a and D are taken functions of the soil profile and the root depth.

2. The coefficient of diffusion

Diffusion must take place through airfilled pores, since the diffusion through water is about 10^4 times as small as that through air. The quantity of airfilled pores in the soil, therefore, is a very important factor. BUCKINGHAM (1904) found a proportionality of D with the second power of the fraction of airfilled pores p. For $p < 0{,}7$ PENMAN (1940) found the relationship $D/D_a = 0{,}66\ p$, where D_a is the diffusion coefficient in air. Penman's observations, however, lie all above $p = 0{,}35$, those of Buckingham above $p = 0{,}25$. On the other hand there are various data from BLAKE and PAGE (1948), BAVER and FARNSWORTH (1940) and TAYLOR (1949) where the diffusion coefficient is practically zero for p ranging from 0,10 to 0,15. The evidence that diffusion practically stops at $p = 0{,}10$ agrees with experiments of WYCKOFF and BOTSET (1936) who found that the hydrodynamical permeability for air is also practically zero if the airfilled pore space becomes less than 0,10.
The values of D/D_a obtained by PENMAN (1940), TAYLOR (1949) and VAN BAVEL (1952) are plotted against p in FIG. 1. Linear adjustment according to VAN UVEN (1946) gives for the relation between D/D_a and p:

$$D/D_a = 0{,}9\,p - 0{,}1 \tag{4}$$

This relation was also used by VAN DUIN (1956) and WESSELING (1957) in their calculations of aeration problems.

3. The production of carbon dioxide

A critical review of all kinds of observations about the production of carbon dioxide is given by WESSELING (1957) and WESSELING and VAN WIJK (1958). ROMELL (1922) states that the CO_2-production in arable land during summer ranges from 0,55 to

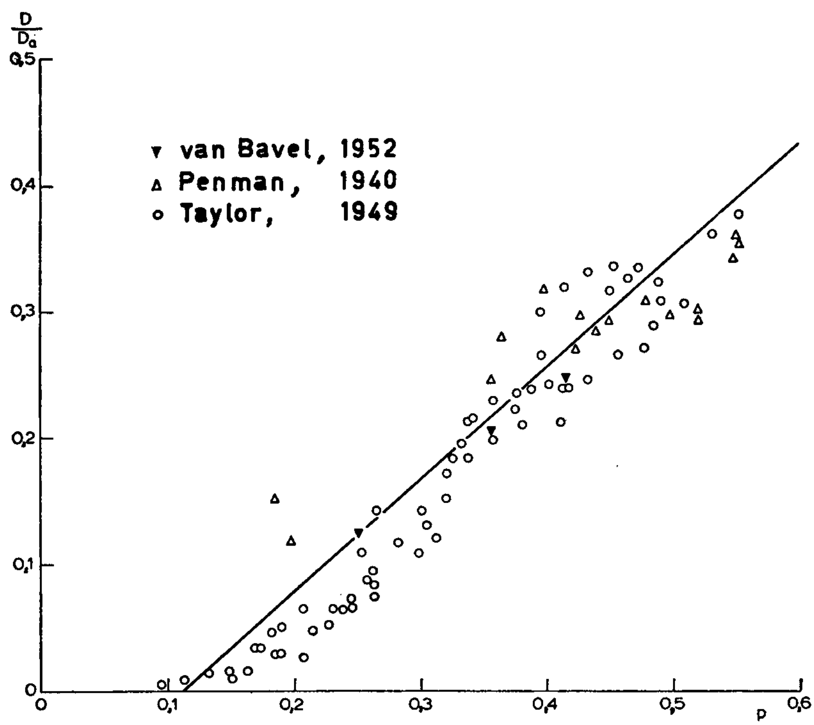


FIG. 1. The relation between D/D_a and the fractions of air-filled pores p after experiments from VAN BAVEL (1952), PENMAN (1940) and TAYLOR (1949)

0,82 g m^{-2} hr^{-1}. KOEPF (1954) gives values of 0,15 to 0,40 g m^{-2} hr^{-1}. WURMBACH (1934) gives for pasture land 8,8 to 11,9 g m^{-2} hr^{-1}. According to LUNDEGAARDH (1924, 1954) temperature is the main factor below 20° C. At higher temperatures the moisture content of the soil will be the limiting factor. Also physical, chemical and biological properties of the soil will influence the uptake of O_2 and the production of CO_2.
For the calculation of the CO_2-content of the soil not only the total production of CO_2 but also the relative production in each layer of the soil will have to be known. In an earlier article (WESSELING, 1957) the present author found that the production for arable land on homogeneous soil profiles may be described by an equation, originally proposed by VAN DUIN (1956):

$$a_z = a_0 \left\{ 1 - \left(\frac{z}{L} \right)^{1/4} \right\} \tag{5}$$

where

a_z = the production of CO_2 in gmol cm^{-2} sec^{-1} at a depth z
a_0 = the production in the top layer
L = the depth at which the production is zero

In eq. 5 the varying amount of roots, bacteria, etc. in the soil will influence the production levels and the zero production depth. At the end of the growing season

L may be taken 90 to 125 cm dependent on the kind of crop. Taking a mean growing rate of the roots of 1 cm per day, the root depth at each moment during the growing season may be computed.

For grassland the production of CO_2 mainly takes place in the upper 20 to 40 cm of the soil and may be taken equal throughout the profile. The same holds for shallow clay layers or humous sandy layers resting on a sandy subsoil in which the roots generally do not penetrate.

4. Solutions of the diffusion equation

ROMELL (1934) gives some solutions of eq. 2 with α constant or linearly decreasing with depth and D constant throughout the profile. Similar solutions are given by VAN BAVEL (1951) for homogeneous and layered soils. Further VAN BAVEL (1951) and VAN DUIN (1956) give solutions in which α does not decrease linearly with depth. The last mentioned author uses eq. 5 for the decrease of α.

In all solutions mentioned above the production of CO_2 was assumed to be restricted to the upper layers of the soil. The constant value of D implies that the solutions are only valid for a constant moisture content of the soil. In the case of grassland or arable land, or soils with shallow rooting properties, the solutions may give a good deal of information about the CO_2-concentration in these soils.

The penetration of roots, however, determines for a great part the possible extraction of water from the soil. In order to get information in how far root development may be restricted by high concentrations of CO_2, diffusion and production of CO_2 in the subsoil has to be taken into account. Then the simplification of a constant moisture content throughout the profile is not permissible, except in the case of sprinkler irrigation. Therefore the present author developed solutions of eq. 2 with a variable diffusion coefficient and α decreasing with depth according to eq. 5 (WESSELING, 1957). These solutions, dealt within the next two sections, are made in such a way that they give the CO_2-content at each depth, when only the total production of CO_2 and the course of the moisture content of the soil profile are known.

5. Aeration in the case of sprinkler irrigation

In general it may be stated that crop production increases with decreasing soil moisture tension except in the case that aeration is the limiting factor. What must now be the minimum pore space of the soil in order to prevent damage from insufficient aeration? In other words, on what kind of soils sprinkler irrigation may be applied with respect to aeration?

The most unfavourable conditions during sprinkler irrigation occur when the soil is kept at field capacity. Taking the soil moisture content constant throughout the profile, the diffusion coefficient is constant. Using now the relation between D/D_a and p given in eq. 3 and a production according to eq. 4, the solution of eq. 2 subject to the boundary conditions given in eq. 3 becomes

$$c = C_0 - \frac{a_0}{0{,}14\,(0{,}9p - 0{,}1)} \left\{ \frac{1}{2} z^2 - \frac{16}{45} z^{9/4} L^{-1/4} - 0{,}2\, L z \right\} \qquad (6)$$

where $D/D_a = 0{,}14$ cm^2 sec^{-1} at 15° C and $C_0 = 0{,}03$ mol %, is the concentration of CO_2 in the air.

The total production of CO_2 will be given by

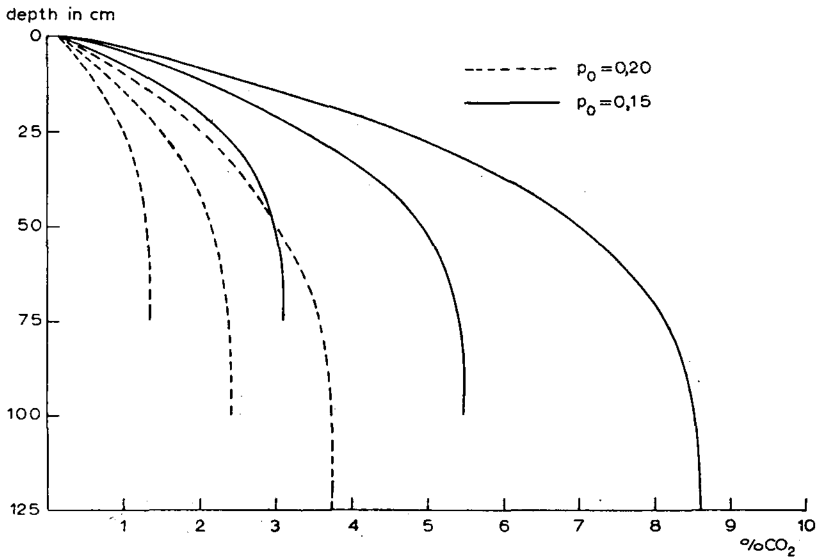
$$\int_0^L a_z d_z \tag{7}$$

With the aid of eq. 5 integration gives

$$\int_0^L a_0 \left\{1 - \left(\frac{z}{L}\right)^{1/4}\right\} dz = \frac{1}{5} a_0 L = B \tag{8}$$

Taking $a_0 = 10^{-6}$ mg cm^{-2} sec^{-1} independent of the root depth L, the course of the CO_2-concentration in the soil according to eq. 6 is given in FIG. 2 for $p_0 = 0{,}15$ and 0,20 and $L = 75$, 100 and 125 cm. For $p_0 = 0{,}20$ and a root depth $L = 75$ cm, the CO_2-concentration at a depth of 15 cm is 0,74 %.

FIG. 2. Course of the CO_2-concentration in the soil in the case of sprinkler irrigation (moisture content is kept constant at field capacity) with $B = \frac{1}{5} a_0 L$ and $a_0 = 10^{-6}$ mg cm^{-2} sec^{-1}. $L = 75$, 100 and 125 cm



In investigations carried out by STOLP and WESTERHOF (1954) where the conditions were in agreement with the above mentioned conditions, aeration was no limiting factor. This is in good agreement with the criterion given by LUNDEGAARDH (1924, 1954) that the CO_2-concentration may not exceed 1 % at a depth of 15 cm. It is, however, by no means sure that this criterion is right for all crops.

If the total production, the root depth and the moisture content at field capacity is known, the CO_2-content of the soil may be derived from FIG. 2 by simply multiplying the concentration found in this figure, by the ratio of the exact CO_2-production and the one that was introduced in this figure.

6. Aeration in other cases

Without sprinkler irrigation the moisture- and air-content of the soil profile depends on the depth of the water table, the kind of soil, the evapotranspiration rate and the root depth of the crop. Generally, the moisture content will increase with depth and consequently the air-content will decrease in this direction. Now it is assumed that the air-content decreases according to

$$p_z = p_0\,(1 - a_k z^k) \tag{9}$$

where the subscripts z and 0 indicate the depth. The value of k is chosen 1 and 2. Further a is given four or five values corresponding with increasing air-gradients in the profile. The corresponding values of L were computed from eq. 9 by taking $L = z$ where $p_z = 0{,}1$, thus the diffusion taken zero when $p \leqslant 0{,}1$.
The values of a, p_0 and L for which solutions are given are collected in TABLE 1 (WESSELING, 1957).

TABLE 1. Values of L computed from eq. 9 for various values of p_0 and a_k. The value of L is the depth where $p_z = 0{,}1$

p_0:	0,20	0,25	0,30	0,35	0,40
$a_1 . 10^3 = 4$	87,5	120,0	—	—	—
6	58,3	80,0	94,4	104,8	112,5
8	—	60,0	70,8	78,5	84,4
10	—	—	56,7	62,9	67,5
$a_2 . 10^3 = 4$	93,5	109,6	118,0	125,0	130,0
6	76,5	89,5	97,4	102,3	106,0
8	66,1	77,5	84,2	88,6	91,9
10	59,2	69,3	75,3	79,4	82,1
12	54,0	63,3	68,6	72,4	75,0

A solution of the problem may be obtained in the following way.
Using eq. 5 for the CO_2-production, we get by substituting the value of B from eq. 8 into eq. 5:

$$\int_0^z a_z d_z = 5\,B\left\{\frac{z}{L} - \frac{4}{5}\left(\frac{z}{L}\right)^{5/4}\right\} \tag{10}$$

By introducing the lower limit $p_z = p_L = 0{,}1$, into eq. 4 we get:

$$D_z = 0{,}9\,D_a\,(p_z - p_L) \tag{11}$$

For $z = L$, eq. 9 becomes

$$p_L = p_0\,(1 - a_k L^k) \tag{12}$$

Substituting now the values of p_z and p_L from eqs. 8 and 11 into eq. 6 and taking $D/D_a = 0{,}14$ cm^2 sec^{-1} (at 15° C) gives

$$D_z = 0{,}126\,a_k p_0\,(L^k - z^k) \tag{13}$$

With the aid of eqs. 19 and 13 the first integration of eq. 2 gives

$$5B\left\{\frac{z}{L} - \frac{4}{5}\left(\frac{z}{L}\right)^{5/4}\right\} = -D_z\frac{\partial c}{\partial z} + D_0\frac{\partial c}{\partial z_0} \tag{14}$$

Because $\frac{\partial c}{\partial z} = 0$ for $z = L$ and $D_L = (0)$ we have from eq. 14:

$$B = D_0 \frac{\partial c}{\partial z_0} \tag{15}$$

and eq. 14 becomes

$$-D_z \frac{\partial c}{\partial z} = B \left\{ 1 - 5\frac{z}{L} - 4\left(\frac{z}{L}\right)^{5/4} \right\} \tag{16}$$

Substituting the value of D_z from eq. 11 into eq. 16 then yields:

$$\frac{\partial c}{\partial z} = - \frac{B\left\{ -1 + 5\frac{z}{L} - 4\left(\frac{z}{L}\right)^{5/4} \right\}}{0{,}126\, a_k p_0 (L^k - z^k)} \tag{17}$$

Integrating eq. 17 with the boundary condition $c = C_0$ for $z = 0$ gives us

$$c = C_0 - \frac{B}{0{,}126\, a_k p_0} \int_0^z \frac{-1 + 5\frac{z}{L} - 4\left(\frac{z}{L}\right)^{5/4}}{L^k - z^k} dz \tag{18}$$

Taking now

$$t = \frac{z}{L} \tag{19}$$

we get for $k = 1$:

$$c = C_0 - \frac{B}{0{,}126\, a_1 p_0} \int_{t=0}^{t=\frac{z}{L}} \frac{-1 + 5t - 4t^{5/4}}{1 - t} dt = C_0 + \frac{BI_1}{0{,}126\, a_1 p_0} \tag{20}$$

and for $k = 2$:

$$c = C_0 - \frac{B}{0{,}126\, a_2 p_0} \int_{t=0}^{t=\frac{z}{L}} \frac{-1 + 5t - 4t^{5/4}}{1 - t^2} dt = C_0 + \frac{BI_2}{0{,}126\, a_2 p_0} \tag{21}$$

The values of I_1 and I_2 are given in TABLE 2.

TABLE 2. Values of I_1 and I_2 from eq. 20 and 21 respectively, for various values of t

t:	0,0	0,2	0,4	0,6	0,8	1,0
I_1	0	0,163	0,274	0,347	0,388	0,401
I_2	0	0,149	0,236	0,284	0,309	0,316

FIG. 3. Course of the CO_2-concentration in the soil according to eq. 20. The air content in the soil decreases linearly with depth. The total CO_2-production $B = 154$ mg m^{-2} hr^{-1}. Figures near curves indicate values of a_1 and p_0 respectively

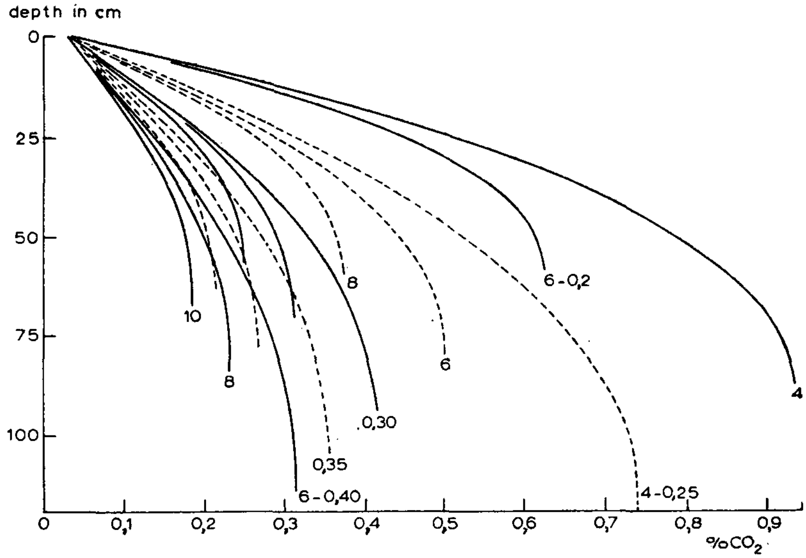
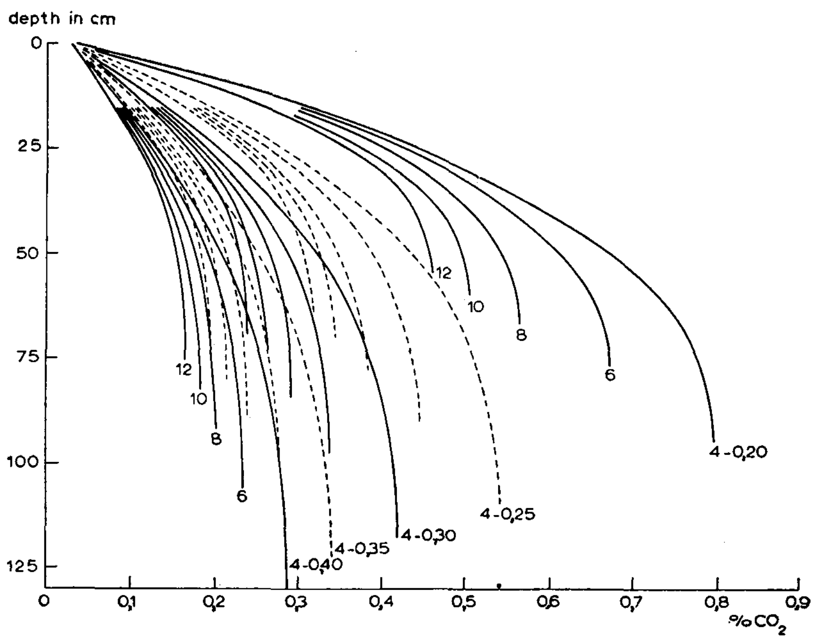


FIG. 4. Course of the CO_2-concentration in the soil according to eq. 21. The air content in the soil decreases with the second power of the depth. The total CO_2-production $B = 154$ mg m^{-2} hr^{-1}. Figures near curves indicate values of a_2 and p_0 respectively



With the aid of the TABLES 1 and 2 and eqs. 19, 20 and 21, the CO_2-concentration can be computed. This has been done for $k = 1$ in FIG. 3 and for $k = 2$ in FIG. 4. In these graphs B was taken 154 mg m^{-2} hr^{-1}. When the total CO_2-production B, the root depth and the course of the moisture content is known, the CO_2-concentration may be determined from these graphs by multiplying the CO_2-content found in the graph by the ratio of the real CO_2-production and that introduced in the graph used.

LITERATURE

BAVEL, C. H. M. VAN — 1951 — A soil aeration theory based on diffusion. *Soil Sci. 72,* 33—46.

—— — 1952 — Gaseous diffusion and porosity in porous media. *Soil Sci. 73,* 91—96.

BAVER, L. D., and R. B. FARNSWORTH — 1940 — Soil structure effects on the growth of sugar beets. *Soil Sci. Soc. Am. Proc. 5,* 45—48.

BLAKE, G. R., and J. B. PAGE — 1948 — Direct measurement of gaseous diffusion in soils. *Soil Sci. Soc. Am. Proc. 13,* 37—42.

BUCKINGHAM, E. — 1904 — Contributions to our knowledge of the aeration of soils. *U.S.D.A. Bur. Soils Bull.* No. 25.

DUIN, R. H. A. VAN — 1956 — On the influence of tillage on conduction of heat, diffusion of air and infiltration of water in soil. (Dutch). *Versl. Landbouwk. Onderz.* No. 62.7.

KOEPF, H. — 1954 — Untersuchungen über die biologische Aktivität des Bodens. *Zeitschr. Acker- und Pflanzenbau. 98,* 289—312.

LUNDEGAARDH, H. — 1924 — Der Kreislauf der Kohlensäure in der Natur. Gustav Fischer, Jena.

—— — 1954 — Klima und Boden in ihrer Wirkung auf das Pflanzenleben. 4e Aufl. Gustav Fischer, Jena.

PENMAN, H. L. — 1940 — Gas and water movement in the soil. *J. Agric. Sci. 30,* 437—462 and 570—581.

ROMELL, L. G. — 1922 — Luftväxlungen i marken som ekologisk faktor. *Medd. från Statens Skoysförsöksanstalt 19,* 125—361.

STOLP, D. W., and J. J. WESTERHOF — 1954 — Agricultural water management. (Dutch). Cultuurtechniek, Alg. Landsdrukkerij. The Hague, p. 20—48.

TAYLOR, S. A. — 1949 — Oxygen diffusion in porous media as a measure of soil aeration. *Soil Sci. Soc. Am. Proc. 14,* 55—61.

UVEN, M. J. VAN — 1946 — Mathematical treatment of the results of agricultural and other experiments. 2nd ed. P. Noordhof N.V., Groningen.

WESSELING, J. — 1957 — Some aspects of the water government in agricultural soils. (Dutch). *Versl. Landbouwk. Onderz.* No. 63.5.

——, and W. R. VAN WIJK — 1958 — Soil physical conditions in relation to drain depth. Drainage Monograph. Chapter V-1. Am. Soc. of Agronomy.

WURMBACH, H. — 1934 — Beiträge zur Kenntnis der Bodenatmung und der Kohlensäurekonzentration in landwirtschaftlich genutzten Flächen. *Wiss. Archiv für Landw. 10,* 484—532.

WYCKHOFF, R. W., and H. G. BOTSET — 1936 — The flow of gas-liquid mixtures through unconsolidated sands. *Physics. 7,* 325—345.