

The seepage of a river into a valley filled with two layers of different permeabilities

L. WARTENA

Research division of the Nederlandsche Heidemaatschappij, Arnhem

Summary

Based on the assumptions of *De Glee* and *Ernst*, formulae have been developed for the seepage from a river in a valley with constant groundwater-level.

For a very wide valley the formulae can be simplified, in which case the formulae developed by Mazure have been obtained.

The results for a wide and a narrow valley filled with soil layers of the same thicknesses and permeabilities have been compared.

1. Introduction

The problem of a river giving seepage into a valley is a problem with which many people are concerned. One case was developed theoretically by MAZURE (1926). His assumptions were that the layer through which the water passes was thick, the potential of the deep groundwater at the beginning of the seepage area was known and the width of this area was infinite.

Making use of the assumptions of DE GLEE (1930) and ERNST (1956) — the latter introduced the radial resistance of the river — it was possible to solve the problem too, in the case of the layer through which the water passes being thin, the width of the valley is so small that the seepage at the end may not be lost sight of and the potential of the groundwater below the river dike in the second layer is unknown.

2. Description and basic formulae

In FIG. 1 one can see for what assumptions the calculations have been made. The rock bottom is horizontal. The first layer B_1 has a bad permeability K_1 , the second layer G_2 has a good permeability K_2 , for which $K_2 > 10 K_1$.

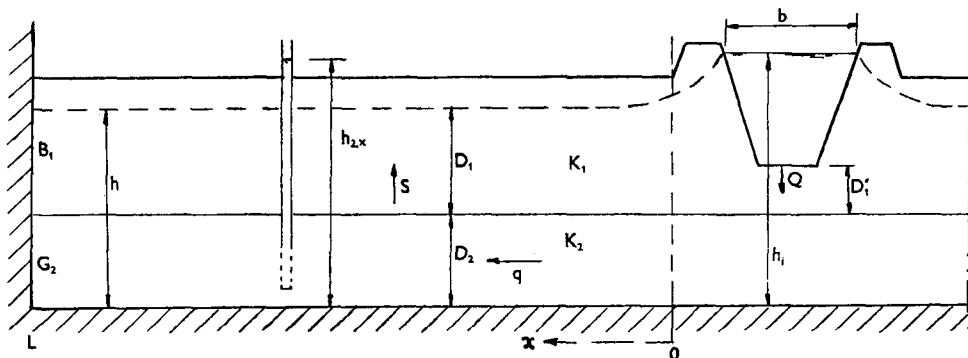
The thickness D_2 of G_2 is constant. The water in the layer B_1 has a constant level h_1 over the bottom of the valley, so the thickness of the water filled surface layer is of a constant thickness D_1 .

In G_2 there is only a horizontal flow and in B_1 only a vertical flow in agreement with MAZURE, ERNST, i.a.

The river bottom is still in B_1 , so the following basic formulae can be put down, when Q is the total amount of seepage from the river to one side. Where the seepage $S = S(x)$, a function of the distance x from the dike and $C = \frac{D_1}{K_1}$, the vertical

Received for publication 2nd March, 1961.

FIG. 1. The impermeable bedrock of the valley is overlain by a good permeable layer G_2 . A less permeable layer B_1 is deposited over G_2 .



resistance of B_1 :

$$S = - \frac{dq}{dx} = \frac{h_{2,x} - h_1}{C} \quad (1)$$

$$q = - K_2 D_2 \frac{d^2 h_{2,x}}{dx^2} \quad (2)$$

3. Solution

Simultaneous solution gives

$$h_{2,x} - h_1 = A e^{x/z_0} + B e^{-x/z_0} \quad (3)$$

$z_0 = \frac{1}{\sqrt{K_2 D_2 C}}$ a characteristic length.

Together with the boundary conditions

as for $x = 0$ we get $q = Q$ and

as for $x = L$ we get $q = 0$

the integration constants A and B can be solved, so we get :

$$q = \frac{Q}{e^{2L/z_0} - 1} e^{(2L-x)/z_0} - e^{x/z_0} \quad (4)$$

In equation (4) the total seepage Q is still unknown.

We can put down

$$Q = \frac{h_j - h_{2,0}}{\gamma} \quad (5)$$

in which

$$\gamma = W + \frac{D'_1}{K_1 b'} \quad (6)$$

W is the radial resistance and $\frac{D'_1}{K_1}$ the vertical resistance between the bottom of the

river and the highly permeable layer. b' is the width of the part of the river out of which water flows, contributing to the seepage Q .

With a decreasing b' one gets a decreasing W .

γ has a minimum value.

The water will flow in such way, that γ is minimal, which value can be calculated by trial and error.

ERNST will shortly be publishing something about the formulae for W in various cases, elsewhere.

Together with the boundary conditions, solution of 3, 4 and 5 gives

$$q = \frac{K_2 D_2 (h_j - h_1) \sinh \{(L - x)/z_0\}}{\gamma K_2 D_2 \sinh L/z_0 + z_0 \cosh L/z_0} \quad (7)$$

$$S = \frac{K_2 D_2 (h_j - h_1) \cosh \{(L - x)/z_0\}}{\gamma K_2 D_2 z_0 \sinh L/z_0 + z_0^2 \cosh L/z_0} \quad (8)$$

$$Q = \frac{K_2 D_2 (h_j - h_1) \operatorname{tgh} L/z_0}{\gamma K_2 D_2 \operatorname{tgh} L/z_0 + z_0} \quad (9)$$

Working with big values of L/z_0 and $(L - x)/z_0$ with hyperbolic functions is difficult and may give use to great errors.

Approximation by means of the formulae 7a — 9a is then preferable.

$$q = Q e^{-x/z_0} \quad 7a$$

$$S = Q e^{-x/z_0/z_0} \quad 8a$$

$$Q = \frac{K_2 D_2 (h_j - h_1)}{\gamma K_2 D_2 + z_0} \quad 9a$$

For $L \rightarrow \infty$ these formulae are exact.

The formulae 7a and 8a were developed by MAZURE (1936) in this form.

4. Discussion

In general the most important thing is to know the seepage at different distances of the river.

It is possible to do this graphically. In FIG. 2 two cases are calculated with the following data

$K_1 = 0,1 \text{ m day}^{-1}$	$D_1' = 2 \text{ m}$
$K_2 = 50 \text{ m day}^{-1}$	$h = 11,5 \text{ m}$
$D_1 = 5 \text{ m}$	$h_1 = 9 \text{ m}$
$D_2 = 4 \text{ m}$	$\gamma = 1,291 \text{ day m}^{-1}$

This, together with $L = 2,000 \text{ m}$, for which 8a and with $L = 200 \text{ m}$, for which formula (8) was used, gave the two curves. Attention must be paid to the K_1 and K_2 . Owing to the fact that in B_1 there is mainly a vertical flow and in G_2 mainly a horizontal flow, the vertical permeability in B_1 and the horizontal permeability in G_2 are of interest.

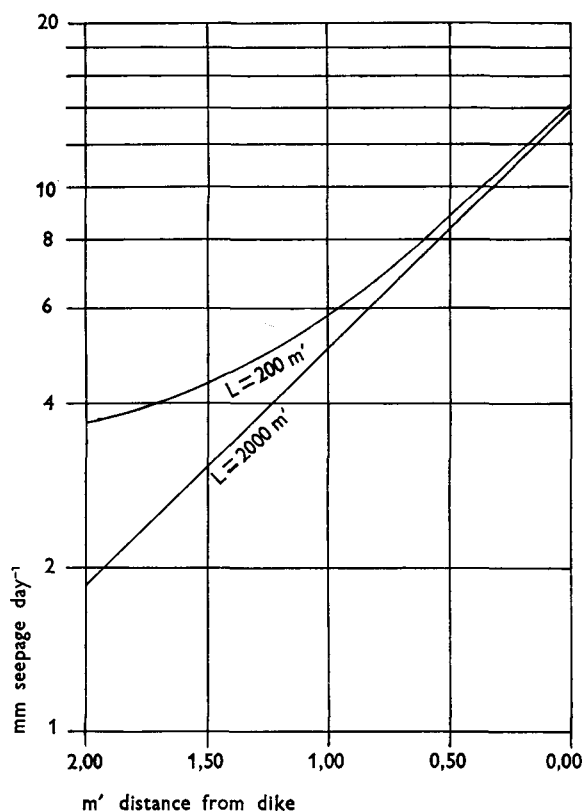


FIG. 2. The seepage from the river into the valley varies with the distance and depends on the width of the valley. For two widths of the valley the relationship is shown. L is the distance from the river to the rock shown in FIG. 1.

Especially when the permeabilities in B_1 are measured by the augerhole method or by a pump test, the event of greater horizontal than vertical permeability, too high values for K_1 are found.

LITERATURE

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