DETERMINATION OF THE CAPILLARY CONDUCTIVITY OF SOILS AT LOW MOISTURE TENSIONS [1]

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SUMMARY

A method is described to calculate the capillary conductivity data from outflow measurements obtained with a suction apparatus also in use for the determination of the lower part of the moisture characteristic of soils. Measurements were made on different soil probes from the same profile. An approximate solution for the influence of the porous plate was introduced. The results of the measurements for a sandy soil and a sandy loam are given.

1 INTRODUCTION

During the last decades the knowledge on the relation between moisture consumption and yield of crops is rapidly increasing. From investigations on this relation it became evident that besides the total uptake of water soil moisture tension is a very important factor in crop growth. This means, that soil moisture tension should be kept low, in order to get maximum yields. Except in the case of sub-irrigation this can only be realized by water transport from the water table into the root zone. Moisture transport only takes place, however, when the water table is kept high or evapotranspiration rates are small. This is due to the fact that the capillary conductivity of the soil is low at high and moderately high moisture tensions. So steep moisture gradients are necessary for moisture flow of some importance (WESSELING, 1957).

The steady state capillary rise of water from a water table can be calculated when the relation between capillary conductivity and soil moisture tension is known. Such calculations are given by WIND (1955), WESSELING (1957) and GARDNER (1958). These authors either used conductivity data found in literature or determined from field experiments, as in the case of WIND. Rather few conductivity data are available, however, despite the fact that a method for the determination has been performed already more than 25 years ago by RICHARDS (1931). During the last few years several new methods were proposed but most of them make use of disturbed soil samples (MOORE, 1939, STAPLE and LEHANE, 1954, BRUCE and KLUTE, 1956, CHILDS and COLLIS-GEORGE, 1950). When conductivity data of a disturbed soil sample are used for the calculation of moisture transport in a well-defined soil profile with its typical sequence of layers, each of them with a pronounced structure, appreciable differences with what actually happens in the profile will be found. The only way out in this case is, to determine the capillary conductivity of each layer of the profile on an undisturbed soil sample.

A theoretical computation of the conductivity data from the pore size distribution i.e. from the moisture characteristic as proposed by CHILDS and COLLIS-GEORGE (1950) has certain advantages. Recently MARSHALL (1958) found a reason-

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able agreement between this theoretical method and experimental results obtained by other authors. The computation takes a lot of time, however, and the so-called matching-factor remains undetermined.

GARDNER (1956) developed a method for the calculation of the conductivity data from pressure plate outflow data. The advantage of this method is that it can be combined with the determination of the higher part of the moisture characteristic of the soil, for which pressure plates are used. A disadvantage is, however, that the method can only be applied at high moisture tensions. As is pointed out above, in flow problems under conditions prevailing in nature, the capillary conductivity of the soil at low moisture tensions is far more important. A method therefore was developed to compute the conductivity at low moisture tensions from outflow data obtained with a suction apparatus that is commonly used for the determination of the lower part of the moisture characteristic. The method, and the results obtained with it, are described in the next sections.

2 DESCRIPTION OF THE APPARATUS

The apparatus, used for the determination of the lower part of the moisture characteristic was modified in such a way that the flow of water out of the soil sample could be measured. The apparatus which was used is given in fig. 1. Undisturbed soil samples (M) are taken in metal rings with an area of 20 cm^2 and 5 cm length. After saturation during two days, the sample was placed on a porous plate (P) which is cemented in a glass-vessel (G), con-

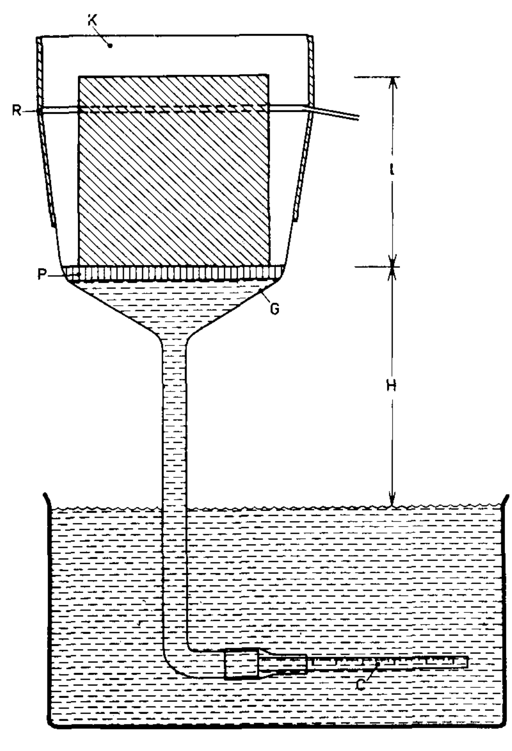

FIG. 1 SUCTION APPARATUS USED FOR THE OUTFLOW MEASUREMENTS. FOR DESCRIPTION SEE THE TEXT.

nected with a water reservoir (W.) The glass-tube ends in a calibrated hori zontal capillary tube (C). The glass-vessel is covered with a cap (K), that is connected with the vessel by means of a rubber-ring (R), in order to prevent evaporation from the sample. A small capillary tube was mounted in order to maintain atmosferic pressure at the upper end of the soil sample.

When equilibrium has been attained between the moisture in the soil sample and a certain suctionforce, the glass-vessel is raised over a small distance, so that the suction increases with a small amount. Due to this increase in suction, water will move to the right hand side, the distance depending on the magnitude of the water removal. By measuring the displacement of the meniscus at different times, the water removal can be determined and from these data the capillary conductivity of the soil may be calculated.

The measurements are susceptible to temperature differences. When room temperature is not constant the meniscus on the capillary tube may become displaced without any change in the moisture content of the soil sample. While the measurements are carried out, with relatively short time intervals, differences in temperature during the experiment can be neglected. The computed values for the conductivity must then, however, be reduced to standard temperature. This standard temperature was taken to be 15° C which is the temperature, occuring in the rootzone of the soil during the greatest part of the summer in the Netherlands.

3 Solution of the flow equation

When equilibrium has been attained at a certain suction of h cm, the moisture content at the upper side of the soil sample will differ only little from that at the lower end of the sample, when the height of the sample is relatively small. Without introducing large errors the moisture content may be considered to be constant throughout the sample. Furthermore, the suctionforce throughout the sample may be taken as a constant, having a value of $h = H + \frac{1}{2} l$, where H is the distance between the water surface in the reservoir and the upper boundary of the porous plate and l is the length of the soil sample.

When the suction is increased from h to $h + \triangle h$ a lower moisture content will be attained after a certain time throughout the sample. When the increase in suction $\triangle h$ is small, there will be only a small difference between the initial and final moisture content in the soil sample and the capillary conductivity is considered to be constant. Moreover $\frac{d\Theta}{dh} = c$, for small changes in moisture content Θ and suction h. For the flow of moisture, the differential equation (1) will then hold true (Gardner, 1958)

$$\frac{\delta\Theta}{\delta t} = \frac{k}{c}\frac{\delta^2\Theta}{\delta z^2} \qquad (1)$$

where

Θ = the part of the volume occupied by water, i.e. the volumetric moisture content of the soil

t = time

k = capillary conductivity

z = length

c = the slope of the moisture characteristic $= \frac{d\Theta}{dh}$

There will be no flow across the upper boundary of the soil sample and henceforth $\frac{\delta \Theta}{\delta z} = o$ at this plane.

Treating the sample as having a length $2\ l$ with its imaginary upper boundary at a constant moisture content Θ_o at $t > o$, the initial and boundary conditions will be:

$$\begin{array}{lll} \Theta = \Theta_n & -l \leq z \leq +l & t = o \\ \Theta = \Theta_o & z = \pm l & t > o \end{array} \qquad (2)$$

The solution of eq. 1, subject to 2, is analogous with that of the equation of heat conduction in a long isolated plate with the ends kept at a constant temperature. This solution is (c.f. CARLSLAW and JAEGER, p. 79 eq. 6).

$$\frac{\Theta - \Theta_o}{\Theta_n - \Theta_o} = \frac{4}{\pi} \sum_{n=o}^{\infty} \frac{(-1)^n}{2n+1} e^{-\frac{k}{c}(2n+1)^2 \frac{\pi^2 t}{4 l^2}} \cos \frac{2n+1}{2l} \pi z \qquad (3)$$

The total quantity of water that will have passed through the lower surface of the soil sample at each time t can be given by

$$Q_t = A \int_{z=o}^{z=l} (\Theta_n - \Theta_o)\, dz \qquad (4)$$

A being the area of the soil sample.

The final amount which can be removed from the soil sample between a suction h and $h + \triangle h$ is

$$Q_T = A\, l\, (\Theta_n - \Theta_o) \qquad (5)$$

With the aid of eq. (3) the relation

$$\frac{Q_t}{Q_T} = 1 - \frac{8}{\pi^2} \sum_{n=o}^{\infty} \frac{1}{(2n+1)^2} e^{-\frac{k}{c} \frac{(2n+1)^2}{4 l^2} \frac{\pi^2 t}{4 l^2}} \qquad (6)$$

can be derived from eqs. (4) and (5). This is the same relation which was used by GARDNER (1956) for the calculation of the capillary conductivity from pressure plate outflow data.

For reasons dealt with in section 5 it is sometimes easier to calculate the conductivity from the slope of the curve Q_t versus t, which gives the outflow velocity at each time t. This quantity is derived from eq. 6 or eq. 3 and is

$$q_t = \frac{2}{l^2} \cdot \frac{k}{c} Q_T \sum_{n=o}^{\infty} e^{-(2n+1)^2 \pi^2 \frac{k}{c} \frac{t}{4 l^2}} \qquad (7)$$

Substituting now eq. 5 in eq. 7 we get

$$q_t = \frac{2}{l} k\, h\, A \sum_{n=o}^{\infty} e^{-\frac{k}{c}(2n+1)^2 \pi^2 \frac{t}{4 l^2}} \qquad (8)$$

4 An approximate solution

For the early stages of flow an approximate solution of the problem under discussion, may be obtained by considering the soil sample as being infinite. The boundary and initial conditions are then given by

$$\begin{array}{lll} \Theta = \Theta_n & z \geq o & t = o \\ \Theta = \Theta_o & z = o & t \geq o \end{array} \tag{9}$$

These conditions are the same as those for infiltration into a semi-infinite horizontal soil column (Klute, 1952) and for drainage to a canal from a semi-infinite strip of soil (Edelman, 1947). The solution of (1) subject to (9) is

$$\frac{\Theta - \Theta_o}{\Theta_n - \Theta_o} = P_F \left(\frac{z}{\sqrt{2 \frac{k}{c} t}} \right) \tag{10}$$

where P_F is the probability integral taken from the tabulated form given by Fisher (Statistical Methods for Research Workers, 1948, p. 77).

The amount of water which has flown out of the sample after a certain time t will be

$$Q_t = (\Theta_n - \Theta_o)\, A \int_o^\infty P_F \left(\frac{z}{\sqrt{2 \frac{k}{c} t}} \right) dz \tag{11}$$

which can be represented by

$$Q_t = 1.14\ A\ (\Theta_n - \Theta_o) \sqrt{\frac{k}{c} t} \tag{12}$$

During the first stages of flow there will be little or no difference between the results of eqs. 3 and 12, with respect to the moisture profile in the soil sample.

As eq. (12) holds only true for a limited length of time, this length of time has to be determined. This is done by using the theory of Krayenhoff van de Leur (1958). Since $1.14 = \frac{2}{\sqrt{\pi}}$ and $c\ h = \Theta_n - \Theta_o$, differentiating eq. 12 with respect to t gives

$$q_t = \frac{1}{\sqrt{\pi}}\, A\, h \sqrt{\frac{kc}{t}} \tag{13}$$

If we take now the largest length of time for which eq. 13 may be used in such a way that the value of q_t from eq. 13 differs less then 1% from the value of q_t according to eq. 8, we get

$$\frac{1}{\sqrt{\pi}} \sqrt{\frac{kc}{t}} \geq 0.99\, \frac{2k}{l} \sum_{n=o}^{\infty} e^{-\frac{k}{c}(2n+1)^2 \pi^2 \frac{t}{4l^2}} \tag{14}$$

Taking now $\gamma = \frac{4}{\pi^2} \frac{cl^2}{k}$ eq. 14 becomes

$$\frac{\sqrt{\pi}}{4 \times 0,99} \sqrt{\frac{\gamma}{t}} \geq \sum_{n=o}^{\infty} e^{-(2n+1)^2 \frac{t}{\gamma}} \tag{15}$$

and for a given combination of c, k and l, t_{max} can be calculated.

5 The influence of the porous plate

The eqs. 8 and 13 derived in sections 3 and 4 are exact, when the porous plate used in the experiments, has no resistance with respect to the flow of water. Generally this is not true and the influence of the sinter may not be ignored, especially when relatively large amounts of water are displaced, thus at large conductivity values of the soil sample, therefore, without introducing the effect of the porous plate, the calculated conductivity data will be too small. On the other hand we failed to find an exact solution in which the effect of the porous plate was accounted for [2]). The outflow data, used for the calculation of the capillary conductivity are, however, taken during very short time intervals. Therefore an approximate solution may be used in which the loss of head in the sinter is related in a linear way with the outflow according to Darcy's law, one can therefore put:

$$q_t = k_p \frac{h_l}{l_p} \tag{16}$$

where

$k_p =$ the hydraulic conductivity of the sinter
$h_l =$ loss of head in the sinter
$l_p =$ height of the sinter.

The hydraulic conductivity of the sinter was obtained by measuring the flow of water through the sinter without soil sample. The area of the plate used for this measurement was equal to that of the soil sample. In order to avoid variations in hydraulic conductivity during the measurement the determinations were made with as short a lapse of time between them as possible. From the known q_t, the loss of head due to the sinter was calculated according to eq. 16 and introduced in eq. 8 and 13 so the hydraulic conductivity was calculated from the equations

$$\frac{q_t}{c} = \frac{2}{l} \frac{k}{c} (h - h_l) A \sum_{n=o}^{\infty} e^{-(2n+1)^2 \pi^2 \frac{k}{c} \frac{t}{4l^2}} \tag{8a}$$

and

$$Q_t = 1.14 \left(\Theta_n - \Theta_o - \Theta_v\right) A \sqrt{\frac{k}{c} t} \tag{12a}$$

where $\Theta_v = h_v \, . \, c$. For q_t and Q_t, the outflow after 1 minute was chosen.

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2) After this article was written Miller and Elrick (*Soil Sci. Soc. Amer. Proc.* 22, 1958: 483–486) gave a method to take into account the influence of the separating medium. An other solution will be given in the near future by Rijtema, also working at our Institute.

6 Results of the measurements

The measurements were carried out for a dune sand containing 1% humus and 99% fraction $> 100\,\mu$, and for a sandy-loam soil ("lichte zavel") containing 1% humus and 1% fraction $> 100\,\mu$, 50% from 50—100 μ and 16% $< 16\,\mu$ (about 10% $< 2\,\mu$).

All outflow measurements gave straight lines for small values of t, when plotted against $\sqrt{t}$. Due to some inertia in the system and the influence of the porous plate this line did not always intersect the t-axis at $t = o$. Nevertheless t was taken in such a way that $t = o$ was at the point of intersection of the line with the t-axis.

In order to simplify the calculation, this form was calculated according to eq. 8a for various values of $\frac{k}{c}$, ($l = 5$ cm; $t = 1$ minute; $h - h_l = 1$ cm;

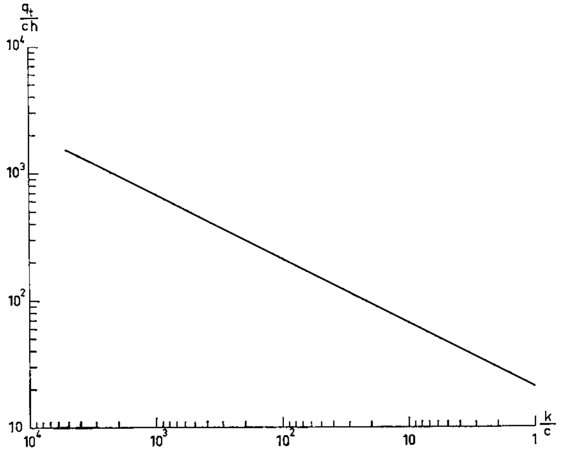

Fig. 2 Values of $q_t/_{ch}$ plotted against $\frac{k}{c}$ for $l = 5$ cm, $t = 1$ min.

$A = 1$ cm^2) giving a straight line when plotted on log : log scales (fig. 2).

The results of the measurements are given in table 1.

For each suction step new soil probes were used taken from the same profile. In the case of the sandy soil, sorption and desorption measurements were carried out with the same probe. For the loamy soil different probes were used for sorption and desorption.

The values according to eq. 12a are somewhat higher than those according to eq. 8a but the difference is small. This is due to the fact, that the time was taken so short that the finite length of the probe did not influence the solution according to eq. 12a.

Because the conductivity values are highly dependent on differences in soil structure as the results show, the error introduced by the approximate solution of the influence of the porous plate will not be very important.

As could be expected, a certain hysteresis effect occured between sorption and desorption values.

Table 1 Capillary conductivity data in cm/day for two soils calculated according to eq. 8a and eq. 12a.

	Dune sand					Sandy loam							
	nr.	eq. 8a		eq. 12a		nr.	eq. 8a			nr.	eq. 12a		
		desorption	sorption	desorption	sorption		desorption	nr.	sorption		desorption	nr.	sorption
suction (cm)	1	52.5		64.5		1	4.82			1	5.80		
5 ⇄ 15	2	7.0		8.4		2	2.42			2	5.39		
	3	40.2		45.9		3	2.54			3	4.40		
	4	37.8		44.1		4	63.1			4	74.7		
	5	32.5		37.6		5	130			5	160		
	6	418		483									
	7	6.2		7.15									
15 ⇄ 20	8	97.2	3.43	110	3.92	6	2.36	30	2.15	6	2.00	30	2.20
	9	1.0	0.678	1.05	0.76	7	5.8			7	6.2		
	10	2.85	0.1123	3.52	0.147	8	18.3			8	32.0		
	11	31.25	0.513	36.0	0.849	9	4.33	31	0.0195	9	3.2	31	0.0216
	12	45.1	0.152	52.0	0.170	10	8.47			10	10.0		
	13	6.9	2.44	7.8	2.88								
	14	36.8		44									
	15	2.79		3.30									
30 ⇄ 50	16	0.163	0.174	0.204	0.217	11	49.2	32	3.4	11	60.0	32	6.25
	17	0.00273	16.4	0.00317	14.0	12	1.18	33	1.36*	12	1.40	33	1.58*
	18	0.056	0.00218	0.0643	0.00225	13	0.067	34	0.108	13	0.0896	34	0.1215
	19	0.0444	0.000202	0.0525	0.000209	14	0.018	35	0.14	14	0.019	35	0.165
	20	–	0.000464		0.0011	15	0.398	36	0.313	15	0.462	36	0.384
	21	–	0.0000678		0.000101	16	12.2	37	4.71	16	14.3	37	5.20
	22	–	0.0166		0.0180								
	23	–	0.000426		0.0009								
80 ⇄ 100	24	0.00198	0.015	0.0072	0.0215	17	0.0255	38	0.005*	17	0.0270	38	0.005*
	25	0.000256	0.00076	0.00054	0.000195	18	0.0215	39	0.000187*	18	0.0144	39	0.000351*
		3 x $\leqslant 10^{-5}$	6 x $\leqslant 10^{-5}$			19	0.0106	40	0.106	19	0.0115	40	0.1955*
						20	0.00026	41	0.00029	20	0.000402	41	0.000385
						21	0.0144	42	0.0078	21	0.0169	42	0.0108
						22	0.22	43	0.0461	22	0.224	43	0.056
						23	0.00013			23	0.00014		
								44	0.00013			44	0.0089
180 ⇄ 300	26	0.00072		0.00126		24	0.0013	45	0.00055*	24	0.00305	45	0.00065*
		5 x $\leqslant 10^{-5}$	5 x $\leqslant 10^{-5}$			25	0.00092			25	0.001075		
						26	0.0344	46	0.00996*	26	0.4750	46	0.00875*
						27	0.000224			27	0.00026		
								47	0.00294			47	0.00325
						28	0.00063			28	0.00125		
							0.00497	48	0.00275			48	0.00135
						29				29	0.095		
							2 x $\leqslant 10^{-5}$		2 x $\leqslant 10^{-5}$				

* Calculated with a value of c equalling the value found during desorption.

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