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WATER TRANSPORT IN DRAINS AS INFLUENCED BY TILE ALIGNMENT [1]

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SUMMARY

Irregularities at the tile joints in a drain line are defined and their effect on the velocity of water in the drain calculated.

The corrections given in figure 4 are usually so small that they can be justifiably ignored in practice.

1 INTRODUCTION

The formula

$$h = \lambda \frac{L}{D} \frac{v^2}{2g} \tag{1}$$

relates pressure drop (h cm) and velocity v (cm/sec) in a pipeline of length L (cm) and diameter D (cm). The gravity constant g is 981 cm sec^{-2} and the dimensionless constant λ is determined experimentally.

In the case of water transport in drain lines other formulas of the same type are also used. Whatever the formula, good results are obtained if correct values are substituted for the empirical constants. The numerical value of these constants (in this case λ) is calculated from measurements on well-aligned drains of uniform tiles.

HEYNDRICKX (1954), reviewing measurements made by EHRENBERG, FEILBERG and ADOLFSSEN of well and badly-aligned drains, concluded that the carrying capacity may be reduced by about 10% in case of bad alignment.

Such values are only of limited use so long as no method is available by means of which the reduction may be calculated from the degree of bad alignment. In this paper methods are indicated for estimating the effect of bad alignment and differences in the diameter of drain tiles on the carrying capacity of water.

2 THE INFLUENCE OF ALIGNMENT ON THE VELOCITY OF WATER IN DRAINS

Applying BERNOUILLI's Theorem and formula (1) to the transport of water in a line of drain tiles gives:

$$h = \lambda \frac{l}{D} v_s^2/2g + Q\, v_s^2/2g \tag{2}$$

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Q is a function of diameter differences between tiles and of displacements at the joints and l is the length of one tile. The index s of v is used to indicate velocity of water in a line with irregularities. Or

$$v_s = \sqrt{\frac{1}{1+\frac{D}{\lambda l}Q}} \cdot \sqrt{\frac{2g}{\lambda} D \frac{h}{l}} = sv \tag{3}$$

in which v is the velocity in the same line without irregularities at the tile joints. The correction factor

$$s = \sqrt{\frac{1}{1+\frac{D}{\lambda l}Q}} \tag{4}$$

accounts quantitatively for the reduction of velocity of water due to diameter differences and displacements of drain tiles in a line.

The dimensionless factor Q in this expression depends solely on the geometry of the joints.

3 The determination of Q

Differences in the cross-section of adjacent tiles, causing sudden expansions and contractions, leads to loss of pressure head. According to Rouse (1950), in the case of expansion loss of pressure head is:

$$h_j = \left(\frac{O_j}{O_k} - 1\right)^2 \frac{v_j^2}{2g} \tag{5}$$

and in the case of contraction:

$$h_k = \left[\left(\frac{O_j}{O_k}\right)^x - 1\right]^2 \frac{v_k^2}{2g} \tag{6}$$

O_j and O_k are cross-sections of the tile with the largest and smallest diameter respectively, and v_j and v_k the velocity of water therein.

Loss of pressure head also results from displacements, d, at the joints of two adjacent tiles (see Figure 1) which again cause sudden contractions or expansions. It follows from Figure 1 and formulas (5) and (6) that in this case the pressure loss is:

$$h_n = \left[\left(\frac{O}{O_n}\right)^{1+x} - 1\right]^2 \frac{v^2}{2g} \tag{7}$$

The value of x has been calculated from the measurements of Weisbach and Von Misen (Rouse op. cit.) on round and square orifices, reproduced in Figure 2.

So long as $1.0 < O/O_n < 1.4$, x equals one with good approximation. No appreciable error is introduced by replacing v_j and v_k by the velocity in a tile of average diameter (v). Thus when two adjacent tiles differ in diameter:

$$Q_r = \left(\frac{O_j}{O_k} - 1\right)^2 \tag{8}$$

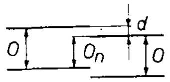

FIG. 1 DEFINITION OF DISPLACEMENT d AND CROSS-SECTIONS O AND O_n.

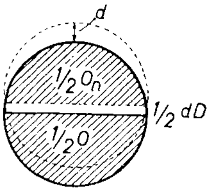

FIG. 3 IF d IS SMALL THE RELATION $\frac{O}{O_n} \simeq \frac{1}{1-\frac{4}{\pi}\frac{d}{D}}$ APPLIES.

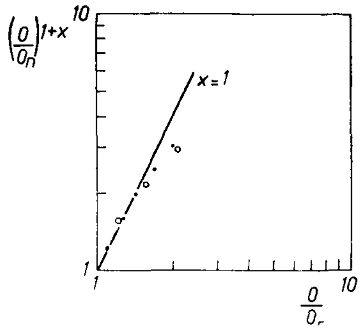

FIG. 2 RELATIONSHIP BETWEEN $(O/O_n)^{1+x}$ AND O/O_n AS CALCULATED FROM OBSERVATIONS OF WEISBACH (•) AND VON MISEN (o) GIVEN BY ROUSE (1950).
THE VALUE OF x IS ABOUT ONE FOR $1 < \frac{O}{O_n} < 1.4$.

It can be seen from Fig. 3 that

$$\frac{O}{O_n} \simeq \frac{\frac{1}{4}\pi D^2}{\frac{1}{4}\pi D^2 - dD} = \frac{1}{1-\frac{4}{\pi}\frac{d}{D}} \tag{9}$$

in which d is displacement as measured in Figure 1.
And with $x \approx 1$:

$$Q_n = \left[\left(\frac{1}{1-\frac{4}{\pi}\frac{d}{D}}\right)^2 - 1\right]^2 \tag{10}$$

The correction s for a drain line with n joints consisting of drains of different diameters, but properly aligned, is:

$$s_r = \sqrt{\frac{1}{1+\frac{D}{\lambda l}\frac{\sum_{k=1}^{n}(Q_r)_k}{n}}} \tag{11}$$

in which $(Q_r)_k$ is the value of Q calculated according to (8) for joint number k.
The correction s for a drain line with n joints not properly aligned but consisting of tiles of the same diameter is:

$$s_n = \sqrt{\frac{1}{1+\frac{D}{\lambda l}\frac{\sum_{k=1}^{n}(Q_n)_k}{n}}} \tag{12}$$

$(Q\ )_k$ being the value of Q for joint number k, calculated according to (10).

4 Calculated and measured corrections

Ehrenberg and Feilberg (Heyndrickx, 1954) compared velocities in two drain lines of uniform tiles, one being properly aligned and the other having given displacements at the joints. Adolfsson (Heyndrickx, 1954) compared the same in two properly aligned drains, one consisting out of uniform tiles and the other containing tiles of an ellipsoidal cross-section.

From these measurements may be obtained an experimental ratio between the velocity in uniform drains and drains with irregularities. The same ratio may be calculated with formulas (11) and (12), as the geometry of the displacements is known.

Results are given in table 1.

Table 1 Values of s_n, as measured by Ehrenberg, Feilberg and Adolfsson in drain lines having known displacements at the joints and as calculated with formula (11) and (12).

	Diameter D (cm)	s_n calculated	s_n measured	$\frac{\text{Calc.}}{\text{Meas.}}$
Ehrenberg	4.90	0.929	0.92	1.01
	8.12	0.960	0.94	1.02
	10.17	0.961	0.95	1.01
Feilberg	5.5	0.935	0.89	1.05
	8.1	0.960	0.96	1.00
	8.1	0.854	0.91	0.94
Adolfsson	5.14	0.924	0.927	1.00
Average		0.932	0.928	0.996

The agreement is such that the calculations are considered accurate enough for all practical purposes.

5 Statistical distribution of irregularities at the joints

The displacement d between two tiles is never smaller than 0 and is usually not greater than a displacement d_{max} allowed by the supervisor. When the displacements are grouped into n classes with class number k $(1;\ 2;\dots;n)$ a width of d, and class boundaries at 0 and kd, the average value of Q is:

$$\bar{Q}_n = \sum_{k=1}^{n} f_k \left\{ \left(\frac{1}{1 - \frac{4}{\pi}\left(k - \frac{1}{2}\right)\frac{d}{D}} \right)^2 - 1 \right\}^2 \tag{13}$$

In most cases the relative frequencies f_k are not known. As d_{max} is small, however, it would not be far off the mark to assume that f_k is $1/n$, so that

$$\bar{Q}_n = \frac{1}{n} \sum_k \left\{ \left(\frac{1}{1 - \frac{4}{\pi}\left(k - \frac{1}{2}\right)\frac{d}{D}} \right)^2 - 1 \right\}^2 \tag{14}$$

in

$$s_n = \sqrt{\frac{1}{1+\frac{D}{\lambda l}\bar{Q}_n}} \qquad (15)$$

As far as diameter differences are concerned, a grouping into n classes with centres at

$$O_1\ ;\ O_2\ ;\ .\ .\ .\ .\ .\ .\ .\ .\ ;\ O_k\ ;\ .\ .\ .\ .\ O_j\ ;\ .\ .\ .\ O_n$$

and with frequencies

$$f_1\ ;\ \ f_2\ ;\ .\ .\ .\ .\ .\ .\ .\ .\ ;\ f_k\ ;\ .\ .\ .\ .\ f_j\ ;\ .\ .\ .\ f_n$$

is possible.

In this case the average Q_r equals

$$\bar{Q}_r = \sum_{j=1}^{n} \sum_{k=1}^{n} f_k\, f_j \left(\frac{O_j}{O_k} - 1\right)^2$$

In case of standarization, the maximum and minimum tolerated diameter are given without specifying a frequency distribution of the diameters themselves. Where there is a sufficient safety margin it may be assumed that $f_j = f_k = 1/n$, giving

$$\bar{Q}_r = \frac{1}{n^2} \sum_j \sum_k \left(\frac{O_j}{O_k} - 1\right)^2$$

Average $\bar{O}$ and class difference $o\bar{O}$ are defined by:

$$O_k = O_o + k\, o\, \bar{O}$$
$$O_j = O_o + j\, o\, \bar{O}$$
$$O_o = \bar{O} - \frac{1}{2} n\, o\, \bar{O}$$

so that

$$\frac{O_j}{O_k} = \frac{1+\left(j-\frac{1}{2}n\right)o}{1+\left(k-\frac{1}{2}n\right)o}$$

As $j = k + i$, with i equal to $0 ; 1 ; 2 \ldots\ldots ; n - 1$, (N.B.: $O_j > O_k$):

$$\frac{O_j}{O_k} = 1 + \frac{i\, o}{1+\left(k-\frac{1}{2}n\right)o}$$

results.

This gives

$$\bar{Q}_r = \frac{1}{n^2} \sum_j \sum_k \left(\frac{i\, o}{1+\left(k-\frac{1}{2}n\right)o}\right)^2$$

Or with good approximation:

$$\overline{Q}_r = \frac{1}{n^2} \sum_j \sum_k (i\,o)^2,$$

provided the diameter differences are not very great.

As

$i = 0$	occurs	n	times
$i = 1$	,,	$2\,(n-1)$	,,
$i = 2$	,,	$2\,(n-2)$	,,
$i = i$	,,	$2\,(n-i)$	,,

and

$i = n - 1$	,,	2	,,

it follows that

$$\overline{Q}_r = \frac{2}{n^2} \sum_{i=1}^{n-1} (n-i)\,(i\,o)^2 \tag{16}$$

in

$$s_r = \sqrt{\frac{1}{1 + \frac{D}{\lambda l}\overline{Q}_r}} \tag{17}$$

6 Some results of calculations

Formulas 14, 15 and 16, 17 can be used to determine the ratios between velocity in a drain line with and without given irregularities.

For this purpose it is necessary to know, the length l, the diameter D and the value of λ as well as the displacements and diameter differences.

As far as l and D is concerned, calculations are restricted to the dimensions as standarized in the Netherlands for drain tiles of baked earth (standard specification 440).

The value of λ depends not only on the dimensions and material of the tiles but also on the Reynolds number of the fluid flowing through. As the effect is not considerable corrections have been calculated only for λ's applicable when h/l is 0.001. (Franke, 1955).

The figures used for calculation are summarized in table 2.

Table 2 Experimental values for λ and $\frac{D}{\lambda\, l}$ (Franke, 1955).

Diameter D cm	Length l cm	λ	$\frac{D}{\lambda\, l}$
5	30.5	0.036	4.59
6	30.5	0.033	6.00
8	30.5	0.030	8.74
8	50.5	$\approx$ 0.030	5.26
10	30.5	0.027	12.25
10	50.5	$\approx$ 0.027	7.36

The results of the calculations are shown in Figure 4.

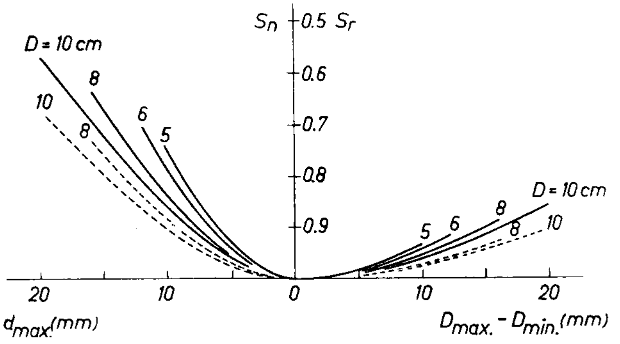

FIG. 4 LEFT-HAND SIDE: RELATION BETWEEN MAXIMUM ALLOWED DISPLACEMENT d_{max} AND CORRECTION FACTOR s_n.
RIGHT-HAND SIDE: RELATION BETWEEN STANDARIZED MAXIMUM AND MINIMUM DIAMETER OF TILE $D_{max} - D_{min}$ AND CORRECTION FACTOR s_r.
The numbers near the curves represent the tile diameter in cm.
—— length of tile 30.5 cm.
- - - - length of tile 50.5 cm.
For further specification of drain tiles see table 2.

On the left-hand side of the figure the relation between correction s_n and maximum allowed displacement d_{max} (in mm) has been given for the drain tiles mentioned in table 2.

The value of s_n decreases with decreasing D, decreasing l and increasing d_{max}. It should be realized that d_{max} is defined as the maximum displacement allowed by the supervisor; it is neither the maximum displacement nor the average displacement. The given definition of d_{max} is the most practical one as it is the only one that can be used when the drain line is designed.

The figure is only valid for the data given in table 2. If other tiles are used and other values of λ are estimated s_n must be calculated directly from formulas 14 and 15.

In the right-hand side of the figure is shown the relation between the difference of the standarized maximum and minimum diameter $D_{max} - D_{min}$ in mm and correction s_r. The corrections are much smaller than on the left-hand side and in most cases negligible.

The combined correction for diameter difference and displacement is always closer to one than

$$\sqrt{\frac{1}{1+\frac{D}{\lambda l}\left(\bar{Q}_n+\bar{Q}_r\right)}}$$

because displacements between two tiles, smaller than the diameter difference of both, have no effect on the velocity of the fluid in the drain.

In the field it is not feasible to distinguish between diameter differences and displacements, and the only practical solution is to interpret all differences at the joints as displacements. In this way the correction is not underestimated, since corrections due to displacements are about twice as large as corrections due to diameter differences.

Any ellipsoidal cross-section of the tiles is not dealt with. Provided the smallest axis of an ellipsoidal cross-section is greater than D_{min} and the longer axis smaller than D_{max}, an ellipsoidal cross-section may be regarded as a circular one having a diameter lying between D_{max} and D_{min}.

Some drain tiles are bent along the centre line, giving rise to unavoidable displacements at the points. Fig. 5 (where d is to be treated as a displacement) shows the most logical method of measuring bends and not straight cuts of the tiles, and of evaluating their effect on the carrying capacity of drains.

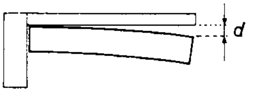

FIG. 5 THE MOST PRACTICAL METHOD OF MEASURING BENDS ALONG THE CENTRE LINE AND NOT STRAIGHT CUTS OF THE TILES. THE DISTANCE d IS TO BE INTERPRETED AS A DISPLACEMENT.

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REFERENCES

FRANKE, A.: Afvoer door drainageleidingen. Rapport N.H.M., Maart 1955.

HEYNDRICKX, G. A.: *Med. Landbouwhogeschool en Opzoekings Inst. Staat van Gent* XIX (1954) 271–309.

ROUSE, H.: Engineering Hydraulics, New York – London, 1950.