of continuity of the heat flux is not satisfied if Eq. (3) is applied tot successive layers. A fuller discussion of the problem will be published elsewhere.

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ON THE TIME DEPENDENCE OF THE EDDY HEAT CONDUCTIVITY IN THE LOWER AIR LAYERS FROM THE RATIO OF THE AMPLITUDES OF THE DIURNAL AND ANNUAL TEMPERATURE WAVES AT 200 AND 10 CM

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SUMMARY

A relation of the type: $a = \alpha (z + z_0)^{\beta}$, in which a is the thermal diffusivity in air, z the height and z_0 a roughness parameter, has been used to calculate the ratio of the amplitudes of the temperature waves at 10 and 200 cm height for values of α ranging from 0.3 to 3×10^3 and values of β ranging from 0.5 to 1.4. It was found that this ratio is only slightly affected by variations in α and β, if $\beta < 0.9$.

Experimental results at Wageningen show that the value of β lies close to 1. No time dependence was found in course of a day or in course of a year.

INTRODUCTION

The FOURIER-equation for the conduction of heat in a semi-infinite, homogeneous medium, in which the temperature, T, is a function of height, z, and of time, t, is:

$$\frac{\delta}{\delta z}\left(\lambda \frac{\delta T}{\delta z}\right) = C \frac{\delta T}{\delta t} \tag{1}$$

The volumetric heat capacity of air, C, is considered as a constant, but the conductivity, λ, besides being dependent upon atmospheric stability, is a function of height. It is assumed that the dependence of λ on height can be represented by (cf. SUTTON (1953) p. 215):

$$\lambda = b\,(z + z_0)^{\beta} \tag{2}$$

The following boundary conditions must be satisfied:

1 at zero height T is a harmonic function of time with amplitude A_0 and angular frequency ω:

$$T\,(o,t) = T_a + A_0\, e^{i\omega t} \tag{3a}$$

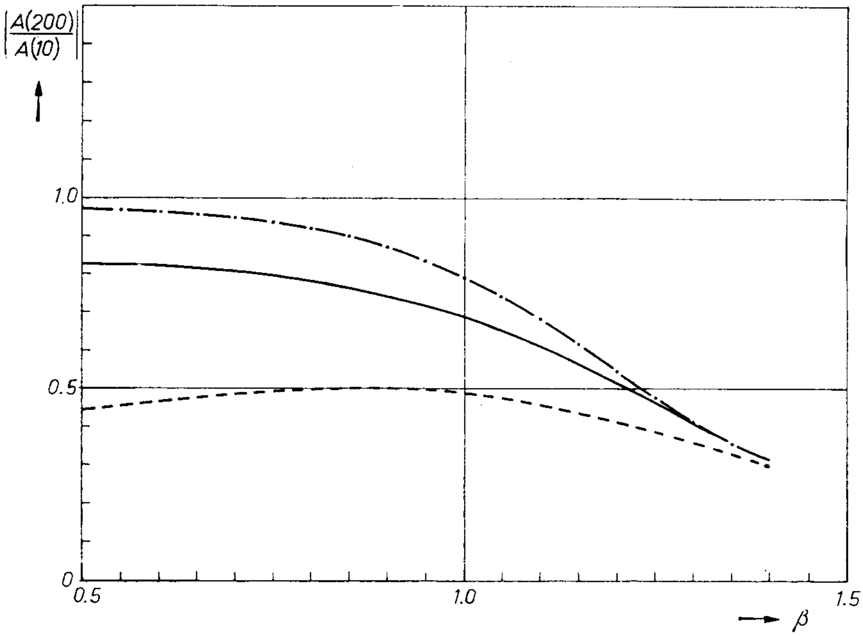

FIG. 1. THEORETICAL RATIO OF THE AMPLITUDES OF THE TEMPERATURE WAVES AT 200 cm AND AT 10 cm HEIGHT AS A FUNCTION OF THE EXPONENT β, THE POWER WITH WHICH THE CONDUCTIVITY AS A FUNCTION OF THE HEIGHT IS SUPPOSED TO INCREASE. THE PHASE DIFFERENCE BETWEEN THE TEMPERATURE WAVES HAS BEEN ELIMINATED.

T_a is an average temperature, independent of height;

2 at infinite height the temperature wave is entirely damped out:

$$\lim_{z \to \infty} T\ (z,t) = T_a \tag{3b}$$

The solution of Eq. (1) with the conditions (3a, b) is (see e.g. JAHNKE and EMDE (1945) p. 146):

$$T\ (z,t) = T_a + A_0 \left(\frac{z + z_0}{z_0}\right)^{\frac{1-\beta}{2}} \frac{e^{p\pi i} J_p\ (\zeta) - J_{-p}(\zeta)}{e^{p\pi i} J_p\ (\zeta_0) - J_{-p}(\zeta_0)}\ e^{i\omega t} \tag{4}$$

in which $J_p\ (\zeta)$ is a BESSEL function with index $p = \frac{1-\beta}{2-\beta}$ and argument

$$\zeta = \frac{2\ (z + z_0)^{(2-\beta)^{1/2}}}{2-\beta} \left(\frac{-i\omega C}{b}\right)^{1/2} \tag{5}$$

while ζ_0 is ζ for the special value of $z = o$ (cf. LETTAU (1952) p. 131).

The ratio of the amplitudes of the temperature waves at 200 cm and 10 cm then becomes:

$$\left|\frac{A\ (200)}{A\ (10)}\right| = \left(\frac{200 + z_0}{10 + z_0}\right)^{\frac{1-\beta}{2}} \left|\frac{e^{p\pi i} J_p\ [200] - J_{-p}\ [200]}{e^{p\pi i} J_p\ [10] - J_{-p}\ [10]}\right| \tag{6}$$

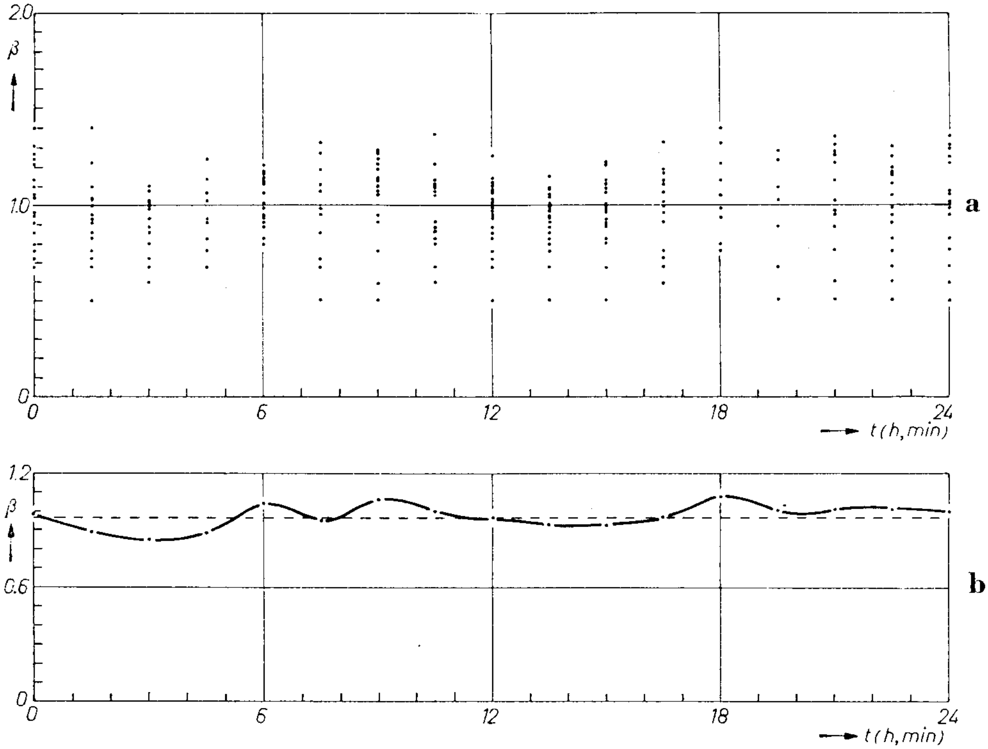

FIG. 2a. VALUES OF THE EXPONENT β OBTAINED FROM EXPERIMENTAL AMPLITUDE RATIOS (PHASE DIFFERENCE ELIMINATED), USING THE CURVE FOR $b = 5 \times 10^{-3}$ OF FIG. 1, PLOTTED VERSUS TIME. TEMPERATURE RECORDS OF 35 DAYS WERE USED, BUT VALUES OF

$$\left| \frac{A\ (200)}{A\ (10)} \right| > 0.83 \text{ OR } < 0.33 \text{ WERE LEFT OUT.}$$

FIG. 2b. AVERAGE β-VALUES FROM FIG. 2a, PLOTTED VERSUS TIME, IN ORDER TO FIND A POSSIBLE DIURNAL VARIATION OF β.

J_p [200] and J_p [10] denoting these BESSEL functions where in the argument (5) z is put equal to 200 and to 10 respectively.

By means of (6) $\left| \frac{A\ (200)}{A\ (10)} \right|$ was computed as a function of β for the following values of the constants :

z_0 , roughnessparameter, 1 cm

ω , angular frequency, 7.27×10^{-5} sec^{-1}

C , volumetric heat capacity of air, 0.29×10^{-3} cal/cm^3 °C

and three different values of the constant b, viz. : 1, 5×10^{-3} and 10^{-4}. The plot is shown in Fig. 1. It is obvious that the amplitude ratio is rather insensitive to the value of b, as a variation of b with a factor 10^4 results only in a variation of $\left| \frac{A\ (200)}{A\ (10)} \right|$ with a factor 2 or less.

Moreover, for values of $\beta < 0.9$ the curves are rather flat, so a variation of β in this region brings about only a small variation of the amplitude ratio.

Discussion of measurements

To obtain the value of β that accounts for the experimental amplitude ratios the temperature records at 200 cm and at 10 cm kept at Wageningen for several years (1948–1954) were used. From these records 35 diurnal records were chosen for which weather conditions were similar. In each case the temperature wave at 200 cm was shifted along the time-axis, so that its maximum coincided with that of the 10 cm wave, in order to eliminate the phase-difference between the temperature waves. The time-axis was devided into 16 equal parts and in each of the deviding points the ratio of the amplitudes was calculated. The values of β, corresponding to these amplitude ratios, were read from Fig. 1, using the curve for $b = 5 \times 10^{-3}$. As this curve is only valid for values of $\left|\frac{A\ (200)}{A\ (10)}\right|$ between 0.83 and 0.33 other values of the amplitude ratio were left out. The values of β thus obtained were plotted versus time in Fig. 2a, while Fig. 2b shows the plot of the average β-values. Finally the values of β were averaged for each of the 35 days and these values were plotted in Fig. 3 versus time.

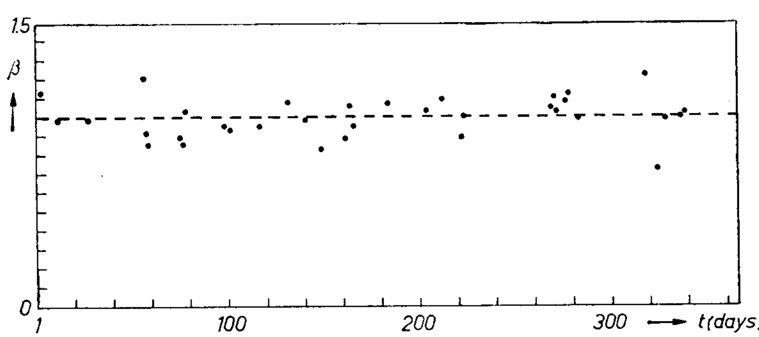

FIG. 3. VALUES OF THE EXPONENT β, AVERAGED OVER EACH OF THE 35 DAYS, PLOTTED VERSUS TIME, IN ORDER TO FIND A POSSIBLE ANNUAL VARIATION OF β.

The values of β, plotted in the Figures 2 and 3, are widely but irregularly scattered about an average value of 0.98, so a time dependence of β if existent at all, does not show up in these results. Apart from minor errors inherent to the measurements and the elimination of the phasedifference between the temperature waves, which is described elsewhere, this wide spread of β is mainly due to the fact that for $\beta < 0.9$ a small variation of $\left|\frac{A\ (200)}{A\ (10)}\right|$ results in a quite large variation of β. So any value of β between 0.82 and 1.12 will fit in with the experimental data.

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