DETERMINATION OF SOIL PERMEABILITY IN SITU [1]

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SUMMARY

The theory of non-stationary flow has been applied to the movement of water in the upper layers of the Pleistocene terrace on the western side of the River IJssel (Fig. 1).

A Fourier analysis of the motion of the water levels in the river and in several wells permitted calculation of the ratio of permeability and storage coefficient of the soil (Fig. 3 and Table 1).

By assuming a storage coefficient of 0.10 and an average thickness of the aquifer of 15 m (Fig. 2), the average permeability (53 m/day) for the area under consideration can be calculated.

It is possible to calculate the discharge per metre without knowing the thickness of the aquifer (0.8 m^3/day). The method can be used to calculate the amount of water needed for irrigation, and the amount of water percolating from the hinterland to the river.

INTRODUCTION

A determination of soil permeability has been carried out in an area near the River IJssel (a tributary of the Rhine) in the Netherlands, with a view to investigating whether irrigation of that area would be economically justified. The reason why this area should be irrigated is that the clay layer of the soil is thin the ground water level low, and therefore the amount of water held by the soil is not sufficient to satisfy the requirements of crops. The method of determining permeability applied here consists in observing the fluctuations of the water level in a number of wells and the delay in time of those fluctuations as compared with fluctuations in the water level of the River IJssel. The same method has previously been used for another area by VAN WIJK and DE WIT (1950). It will be described here in full since no account of it has yet been published in English.

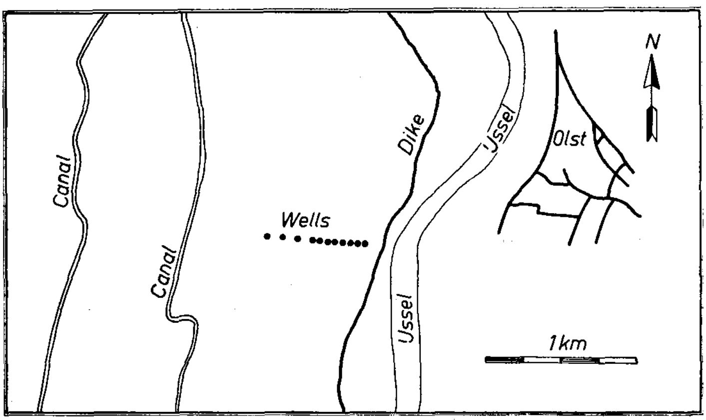

FIG. 1. SKETCH MAP OF THE AREA.

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Description of the area

The area concerned is shown in Fig. 1. Drillings to approx. 30 m have revealed that the lowest layers consist of coarse sand, and that the higher layers are composed of sandy and loamy material, medium coarse sand, river sand and river clay. At about 15 m below the surface there is, in most cases, a loamy layer, the permeability of which is very low.

It is highly probable that the movement of the ground water is restricted to the soil above this layer (see Fig. 2).

In these soils the ground water level shows great fluctuations. During this investigation the water level of the IJssel fluctuated from 5.13 m + N.A.P. [2]) to 0.83 m + N.A.P. In summer the ground water level may be 3 m below the surface, and in winter approx. 0.20 m below it. This situation is exactly counter to the needs of agriculture.

In order to obtain data about the fluctuations of the water level in the soil the "Government Service for Utilization of Land and Water" [3]) drilled 11 wells in a straight line perpendicular to the river, at distances of 50 and 100 m apart. The first well is situated 180 m from the river (Fig. 1). The water levels in the wells were measured daily.

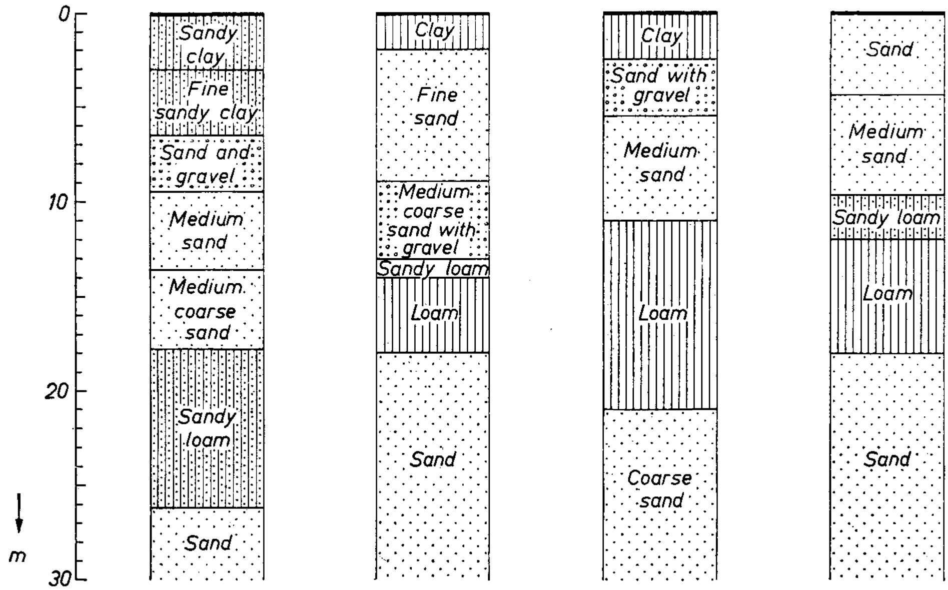

Fig. 2. Some profiles of the region.

Determination of permeability

The usual methods of determining soil permeability consist in:

1 Applying Darcy's law to the homogeneous flow of water through a sample of the soil.

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[2]) h.m. + N.A.P. is used in the Netherlands to indicate a height of h meters above the average high water level of the sea in summer at Amsterdam.

[3]) The present study was carried out at the request of the Cultuurtechnische Dienst (Government Service for Utilization of Land and Water). The author is indebted to the head of the research section of that body, Mr. W. C. Visser, for information concerning the local situation and the plans for land amelioration in the region investigated.

2 Using ZUNCKER's formula (or similar formulae). ZUNCKER's formula is a method of calculating permeability from the pore space, specific surface and characteristic diameter of the soil particles.
3 Using the auger hole method (HOOGHOUDT (1936), KIRKHAM (1945), an experimental determination based on the flow towards a well in which the level of the water is lowered artificially.

The well-known drawbacks of these methods are:
1 A sample is often not representative of the soil, owing to disturbance caused by sampling.
2 Reliable results are obtained only for coarse, sandy soils.
3 Information is only obtained about the immediate neighbourhood of the hole.

The determination described in this paper is based upon non-stationary movement of soil water. An average value of the permeability of undisturbed soil in the entire area is derived from it.

DESCRIPTION OF THE METHOD

The water level in a well situated at some distance from a river responds in course of time to fluctuations in the level of the river. The underground flow of water from well to river or vice versa depends on the gradient of the ground water level, and on the permeability and capacity for storing water of the soil layers through which flow takes place. If a case is considered involving a river flowing in a straight line and a homogeneous soil, the following differential equation is obtained for the height of the water level at a certain distance, in a direction perpendicular to the river, at a certain time t.

$$K \cdot \frac{\delta^2 h}{\delta x^2} = C \frac{\delta h}{\delta t} \qquad (1)$$

where K = permeability, which is assumed to be constant,
C = storage factor, i.e. number of cc water taken up by 1 cc of the layer when water pressure rises 1 cm. C is also assumed to be constant,
h = height of water level,
x = distance from the river in a perpendicular direction.
t = time.

This equation is valid when the fluctuations of the ground water level are small in proportion to the thickness of the aquifer. As the fluctuating water level of the river can be approximately evaluated by a trigonometric expression, from a certain instant $t = 0$ onwards, the simplest solution that needs to be considered is a cosine or a sine function for $x = 0$, i.e. at the river.

The exact solution of eq. 1 applicable to the water movement in the soil consists, then, of three parts. Firstly, a trigonometric expression by which the fluctuations in the river from $t = 0$ are taken into account. Secondly, a linear term corresponding to the slope of the average ground water level, which causes the continuous drainage of the hinterland towards the river. Thirdly, a correction for the deviations of the actual ground water level from those calculated by the trigonometric expression at the zero point of the time scale $t = 0$.

This last term must be added because the fluctuations of the river are not strictly periodic, and thus the trigonometric expression is not in general an approximation to fluctuations previous to that instant, although it has been so chosen as to approximate fairly closely to fluctuations after $t = O$. (This correction will be considered at a later stage).

Periodic solution

The solution of eq. 1 that becomes transformed in to cos $n\omega t$ for $x = 0$ is

$$h = e^{-x/D} \cos (n\omega t - x/D) \qquad (2)$$

in which $n = 1,2,3 \ldots\ldots$ any positive whole number.

The constant $D_n = \sqrt{\frac{2p}{nw}}$ is a length characteristic of the soil; it depends on the circle frequency ω of the cosine and on $p = K/C$.

p will be called "diffusivity", by analogy with the theory of diffusion, in which the same differential equation occurs.

$\omega = 2\pi/T$, T being the period during which the motion takes place; ω is often called the circle frequency.

x = distance from the river in a direction perpendicular to it.

According to eq. 2 fluctuations of the river ($x = 0$, for the river) are attenuated at a distance x by a factor e^{-x/D_n}, and a phase shift $-x/D_n$ occurs at that distance.

The actual height of the water level of the river as a function of time is then expanded in a Fourier series according to one of the standard procedures described in common textbooks. One obtains:

$$b(0,t) = h_{av} + \sum_{1}^{\infty} h_n \cos (n\omega t - \varphi_n) \qquad (3)$$

in which $h(0,t)$ is the height of the water level of the river, ($x = 0$) at time t, h_{av} is the average height during the interval of time under consideration, and h and φ numerical constants which are found from the Fourier analysis.

The water level in a well situated at a distance x from the river becomes:

$$h(x,t) = h_{av} + Rx + \Sigma\, h_n e^{-x/D_n} \cos (n\omega t - \varphi_n - x/D_n) \qquad (4)$$

The constant R is characteristic of the magnitude of the slope of the ground water level which causes, on the average, drainage of the hinterland towards the river. It is small in the case under examination.

In the present problem the period November 1952 until June 1953 has been considered, in which period a high winter and a low summer level occurred (Fig. 3). It contains 32 weeks and if, as is the case in the following pages, time is measured in weeks, $\omega = 2\pi/32$. This value must be inserted in eqs. 3 and 4. The trend of the water levels of the River IJssel, and of the first, second, fifth and eighth wells, and the rainfall, are given in Fig. 3.

Fourier expansion

The Fourier expansion for the motion of the water level of the IJssel has been found to be as follows, up to and including the 4th term:

$$h(0,t) = 2.32 + 1.22 \cos \left(\frac{2\pi t}{32} - 1.54\right) + 0.66 \cos \left(\frac{4\pi t}{32} - 1.74\right)$$

$$+ 0.23 \cos \left(\frac{6\pi t}{32} - 0.35\right) + 0.43 \cos \left(\frac{8\pi t}{32} - 0.80\right) \qquad (5)$$

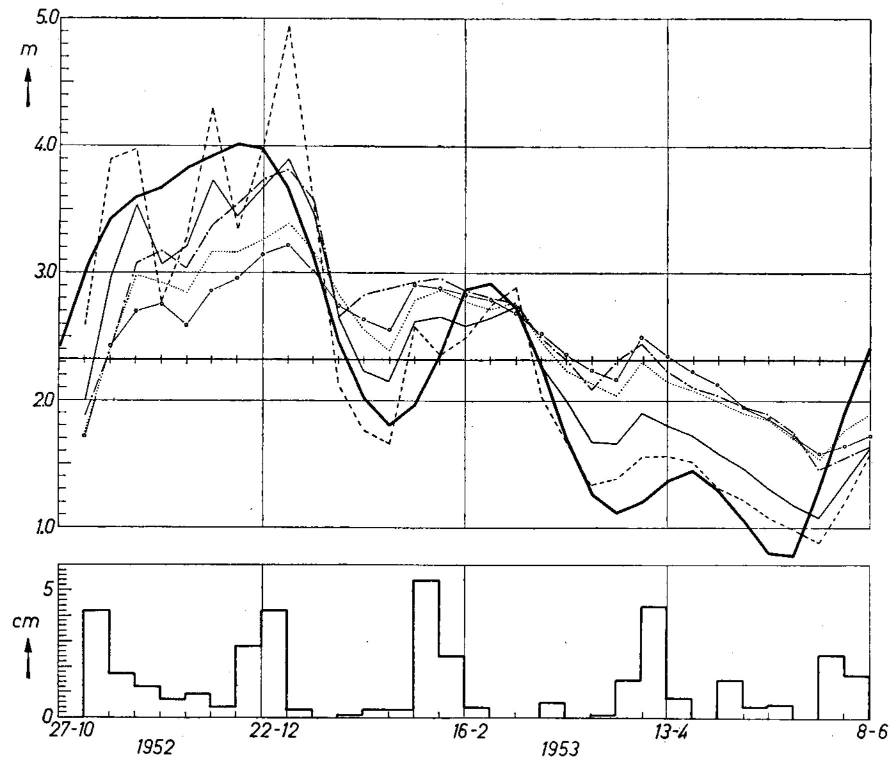

FIG. 3. TREND OF WATER LEVELS.
In the River IJssel.
In the first well (180 m).
In the second well (230 m).
In the fifth well (380 m).
In the eighth well (530 m).
Representation of the River.
IJssel by four Fourier orders.
Rainfall (week).

The representation of the River IJssel by four Fourier terms (Fig. 3) shows deviations of the order of the second and higher terms from the actual fluctuations: consequently, these terms are less reliable. Secondly, the amplitude of the periodic Fourier terms decreases with increasing order n. Accordingly, the first term is the most suitable one for a calculation of $p = K/c$; next comes the second term; and so on. Thirdly, if p is not strictly constant, but varies with the distance x, in theory the terms of higher order would rapidly become unsuitable for the determination of p. Fourthly, as eq. 2 shows, the amplitude of a term decreases more quickly with distance the higher n is, and these terms therefore become less important the farther away the well is. Finally, the correction for the deviation of a true periodic motion of the water level has been found to be such that, for that reason alone, the third and fourth terms ($n = 3,4$) cannot be used at all for calculation of p. Thus, a calculation of p from the first and second terms only will produce the best results.

In this paper we have calculated a value of p from the terms in which

$n = 1$, $n = 2$, respectively, but far more weight has been given to the results obtained from the first term than to those from the second.

The Fourier expansions up to the second term have been obtained in respect of all wells. By way of example, that of the first well $x = 180$ m is given in eq. 6:

$$h(180,t) = 2.36 + 1.00 \cos \left(\frac{2\pi t}{32} - 1.70\right) + 0.51 \cos \frac{4\pi t}{32} - 1.89) \qquad (6)$$

Two independent determinations of p can be obtained from each term in the expansion of any well – one from the ratio of its amplitude to the amplitude of the corresponding term in the expansion for the River IJssel (eq. 3), and the other from the shift in phase, e.g.:

$$\ln h_n(0,t)/h_n(x,t) = x/D_n = x/\sqrt{\frac{2p}{n\omega}} \qquad (7)$$

$$\text{phase shift} \qquad = x/D_n = x/\sqrt{\frac{2p}{n\omega}} \qquad (8)$$

The values of $\ln h_n(0,t)/h_n(x,t)$ and of the phase shifts from the Fourier expansions of the first terms in relation to the distances of the wells from the river are shown in Fig. 4a; Fig. 4b shows the same in respect of the second terms. A straight line which fits these points has therefore been drawn by hand. The gradient of this line expressed as tg φ is equal to $1/D_n$, and hence an average value of p can be calculated from D_n according to formulae 7 and 8.

In Fig. 4a there appears to be no systematic difference between the points resulting from amplitude ratios and phase shifts, respectively.

The points resulting from the phase shifts of the second terms appear to lie systematically lower than those resulting from the amplitude ratios. Two

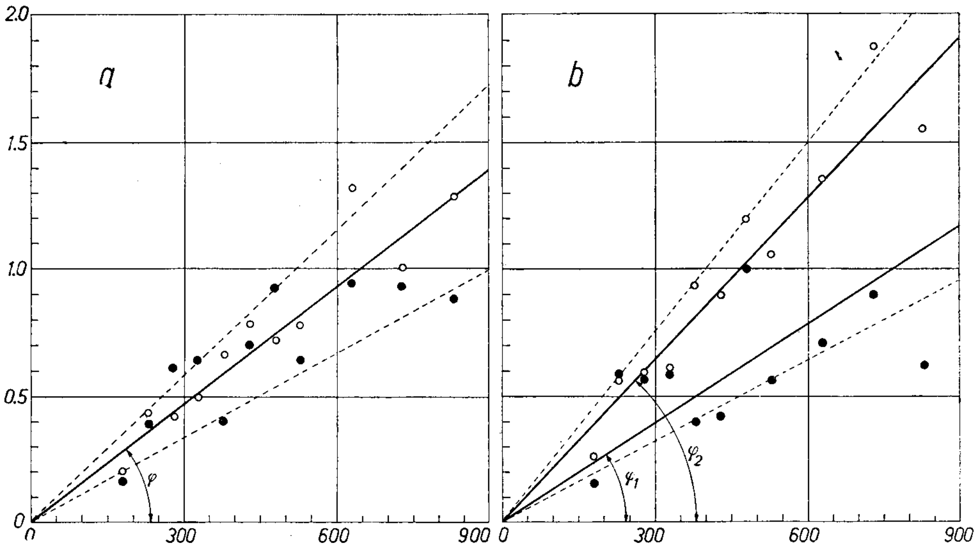

FIG. 0a, b. THE VALUES OF $\ln h(0,t)/h(x,t)$ (o) AND PHASE SHIFTS IOJ FROM THE FOURIER EXPANSIONS OF THE FIRST AND SECOND TERMS (4a AND 4b, RESPECTIVELY) IN RELATION TO THE DISTANCES OF THE WELLS FROM THE RIVER. THE DOTTED LINES SHOW THE UPPER AND LOWER LIMITS.

different lines have therefore been drawn. The upper and lower limits of p for both terms have also been calculated (dotted lines in Figs. 4a and b).

Correction for non-periodicity

Before proceeding to a general discussion of results, the correction for non-periodicity, which up till now, has been ignored, will be considered. Let $h(x,o)$ be the value of eq. 4 at $t = 0$ and let $f(x)$ denote the actual height of the ground water at x and when $t = 0$. If then $X_{(n)}$ denotes the difference $f(x) - h(x,o)$ the correction function $u(x,t)$ that has to be added to eq. 4 is (Steggewentz, 1933):

$$u = \frac{1}{\sqrt{\pi}} \left\{ \int_{\frac{-x}{2\sqrt{pt}}}^{\infty} X(2\zeta\sqrt{pt} + x)\, e^{-\zeta^2}\, \delta\zeta - \int_{\frac{+x}{2\sqrt{pt}}}^{\infty} X(2\zeta\sqrt{pt} - x)\, e^{-\zeta^2}\, \delta\zeta \right\} \quad (9)$$

The value of u is of the same order as the third and fourth terms of the Fourier series when $t = 8$ weeks. So the third and fourth terms cannot be used at all for the calculation of p.

Discussion of results

In the following table values of p obtained from $1/D_n$ of the first and second terms are given.

n	ampl. ratios	phase shifts	upper limit	lower limit
1	0.60×10^4 m^2/day	0.60×10^4 m^2/day	1.15×10^4 m^2/day	0.38×10^4 m^2/day
2	0.61×10^4 m^2/day	1.70×10^4 m^2/day	2.50×10^4 m^2/day	0.46×10^4 m^2/day

The deviations in the phase shifts of the second terms are of the order of one week, i.e., of the same order used for calculation of the Fourier terms.

By giving p from the first term a weight twice that of p from the second term, an average value of 0.80×10^4 m^2/day is obtained.

In the case of this soil, a value of 0.10 was estimated for the storage coefficient (the amount of water needed to salurate that part of the soil situated directly above the capillary zone).

A depth of 15 m was assumed for the layers in which most of the water movement takes place (m) (see the description of the area).

The permeability can now be calculated. We have $K = p \times C = 0.80 \times 10^4 \times \frac{0.10}{15} = 53$ m^2/day.

The discharge per metre is $Q = K \times m \times I$, in which $I = \delta h/\delta x$ or $Q = p \times \frac{0.10}{m} \times m \times I = 0.10\, p \times I$.

The thickness m does not appear in the final expression for the discharge per metre length. In this case I is 0.001. So $Q = 0.10 \times 0.80 \times 10^4 \times 0.001 = 0.8$ m^3/day.

For D, a value of 760 m was obtained. For example, at a distance of 760 m from the river the amplitude is $e^{-1} \times$ the amplitude of the river; or $0.37 \times 1.22 = 0.45$ m.

At a distance of 1520 m the amplitude is $0.135 \times 1.22 = 0.16$ m.

The effect of the river decreases strongly with increasing distance, and by the time a distance of even 1000 m from the river has been covered, so many other influences (drainage canals, discharge from the hinterland, rainfall) have played a part that the effect of the river on the ground water level becomes untraceable.

The permeability value calculated from the non-stationary water movement constitutes exactly the average value for this area which is needed for irrigation calculations, since it is determined from actual transport of water over a considerable distance. Regions of high permeability, which would probably escape attention in the case of soil sampling or auger hole testing, now automatically contribute to the final average value.

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List of symbols and units

Symbols		Units
C	– storage factor	1/m
D	– constant characteristic of the soil	m
e	– base of natural logarithms	–
h_{av}	– average height of water level	m
$h(x,t)$	– height of water level at distance x and time t	m
I	– average slope of water level	–
K	– permeability	m/day
m	– thickness of the aquifer	m
n	– any positive whole number	–
p	– diffusivity	m^2/day
R	– average slope of ground water level	–
t	– time	day, week
u	– correction function	–
x	– distance from the river in a perpendicular direction	m
X	– difference between actual height and calculated height of the ground water level when $t = 0$	m
ω	– circle frequency	1/week