SOME CONSIDERATIONS ON THE DIURNAL VARIATION OF TRANSPIRATION [1]

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ABSTRACT

The diurnal variation of the measured transpiration from pots with alfalfa plants and the readings of several evaporimeters for a case reported by Briggs *and* Shantz *are compared with the calculated values according to* Penman's *theory. The influence of radiation on transpiration and evaporation is clearly illustrated (fig. 1). The ratio between transpiration and calculated evaporation is almost constant and shows less variation than the transpiration/evaporimeter-reading ratios (fig. 2). The differences in magnitude between the transpiration and the evaporation values and the limitations of the theoretical treatment are discussed. It is pointed out that the evaporation from isolated plants or groups of plants which project above their surroundings, will usually be greater than the evaporation from a closed homogeneous vegetation with the same height as these plants.*

List of symbols and units

Symbol		Unit
A	amplitude of diurnal variation of temperature	°C
C	heat capacity per unit volume of soil	cal/cm³ °C
E	evaporation or transpiration	mm/h
E_0	evaporation from a water surface	mm/h
$\underline{E'_0}$	corrected value of E'_0	mm/h
E_T	auxiliary quantity defined by equation (4a)	mm/h
E_a	transpiration and evaporation from a surface covered by vegetation	mm/h
e_a	vapour pressure in the air	mmHg
e_s	vapour pressure at the surface	mmHg
f	factor defined by equation (2)	$mmH_2O/mmHg$
H	net gain of radiation energy at the surface	cal/cm² h
K	heat flux into the air	cal/cm² h
L	heat of vaporization of water	cal/g
S	heat flux into the soil	cal/cm² h
T	temperature	°C
T_a	air temperature	°C
T_s	temperature at the surface	°C
$\overline{T}_s$	average temperature at the surface	°C
t	time	h
u	wind velocity	m/sec
u_2	wind velocity at height of 2 m	m/sec
γ	psychrometer constant	mmHg/°C
Δ	slope of saturation vapour pressure curve	mmHg/°C
ε	saturation vapour pressure	mmHg
ε_a	saturation vapour pressure in the air	mmHg
ε_s	saturation vapour pressure at the surface	mmHg
λ	thermal conductivity of soil	cal/cm h °C
τ	period of diurnal temperature wave	h

1 Introduction

In recent years the computation of evaporation and transpiration starting from physical principles has received much attention. These principles can be placed under two headings, viz. a) the conservation of energy, b) the turbulent

1) Received for publication December 16, 1952.

exchange of water vapour and of heat. Among the contributions in this field we mention especially the well-known work of PENMAN (1948), who derived a formula based on both principles for the computation from standard meteorological observations of the evaporation from a water surface and from a cropped land surface with unlimited water supply. The period for which these calculations are performed can be one day, but it usually covers several days or weeks.

In ecological studies much importance is ascribed to the diurnal variation of transpiration. The diurnal variation of evaporation is usually determined by the aid of an evaporimeter. The *Piche* evaporimeter is for instance in widespread use for this purpose, abroad and in the Netherlands. It therefore seemed justified to extend the physical considerations to the computation of the daily trend of evaporation. Although we are wide away from a complete theory of this phenomenon, some important principles and facts can be demonstrated by the application of the above mentioned physical principles. By way of illustration we treated a case described by BRIGGS and SHANTZ (1917), who compared the transpiration of pots with alfalfa plants with the readings of various types of evaporimeters.

After a brief outline of the principles underlying PENMAN's theory is presented, this theory will be applied to compute the diurnal variation of the evaporation from the data of BRIGGS and SHANTZ. The results of the calculations will be compared with the measured transpiration and with the readings of a number of evaporimeters. Finally the limitations of the theoretical treatment will be discussed.

2 Physical theory

Application of the principle of the conservation of energy to the ground surface leads to the following formula:

$$(1) \qquad 0.1\,LE = H - K - S,$$

where E = the evaporated water per unit time, L = the heat of vaporization of water, H = the net gain of radiation energy per unit time at the ground surface, K = the heat flux into the air and S = the heat flux into the soil. E will be expressed in mm/h; L in cal/g; H, K and S in cal/cm^2 h. Therefore the factor 0.1 (with the dimension g/cm^3) must be introduced in the left hand side of (1).

Application of the theory of turbulent exchange to the atmosferic layer adjacent to the earth surface gives rise to the following equation:

$$(2) \qquad E = (e_s - e_a)\,f,$$

where e_s = the vapour pressure at the earth surface, e_a = the vapour pressure at a given height above the surface, usually taken as the standard height of meteorological observations, e.g. about 2 m, f = a function of the wind velocity profile. f depends on the average wind velocity, u, at a given height, on the aerodynamic properties of the surface and on the conditions of thermal stability in the air layer under consideration.

The same theory further leads to the following relation between E and K:

$$(3) \qquad \frac{K}{0.1\,LE} = \frac{T_s - T_a}{e_s - e_a}\,\gamma,$$

where T_s = the temperature at the surface, T_a = the temperature at the height where the vapour pressure equals e_a and γ = the psychrometer constant. In deriving (3) it is assumed that the exchange of heat and of water vapour in the air can both be described by the same "turbulent diffusivity factor" (cf. RIDER and ROBINSON (1951)).

By neglecting the term S in the right hand side of (1) and combining (1), (2) and (3) PENMAN arrived at the following formula for the evaporation from a water surface, E_0 :

(4) $0.1\ LE_0 = (\Delta H + \gamma E_a)/(\Delta + \gamma)$, with

(4a) $E_a = 0.1\ Lf\ (\varepsilon_a - e_a)$,

(4b) $\Delta = (\varepsilon_s - \varepsilon_a)/(T_s - T_a) \approx (d\varepsilon/dT)_{T = T_a}$,

where ε = the saturation vapour pressure.

PENMAN gives the following empirical expression for f:

(5) $f = 0.0146\ (1 + 0.54 u_2)$.

Here u_2 = the average wind velocity at a height of 2 m, expressed in m/sec. E_a is again measured in mm/h, while the vapour pressure is expressed in mmHg.

In equation (4) the factors referring to the surface, which are usually unknown and, moreover, are difficult to measure, are eliminated and replaced by the analogous quantities for standard height of observation. Since the quantity S is usually small as compared with $H - K$ over a period of one day or several days, the daily evaporation can be computed from standard meteorological observations and radiation measurements. If the latter are not available the quantity H can also be computed from meteorological measurements by empirical formulae, for which we refer to PENMAN's paper.

PENMAN found experimentally, that the evaporation and transpiration from a surface covered with short grass, E_T, is smaller than E_0. The ratio E_T/E_0 varied from 0.8 in summer to 0.6 in winter. PENMAN ascribes this phenomenon to the influence of the resistance to vapour diffusion located in the stomata, to the influence of the closing of the stomata during part of the 24 hours of a day and to the fact that a surface covered by vegetation reflects a larger part of the incoming radiation than does a water surface (cf. PENMAN and SCHOFIELD (1951)).

Other investigators found under certain circumstances ratios $E_T/E_0 > 1$, especially in a vegetation with larger vertical dimensions. This point will be briefly discussed in section 4.

If the quantity S in equation (1) is not negligible the quantity H in equation (4) must be substituted by $H - S$. In order to make an estimate of S we assumed that the temperature at the surface can be expressed as a sinuoidal function of the time, t, hence:

(6) $T_s = \overline{T}_s + A \sin 2\pi t/\tau$,

where the period τ equals 24 h if we consider the daily variation of T_s. If the soil is homogeneous it can be shown that S is represented by:

(7) $S = A\ (2\pi\lambda C/\tau)^{1/2} \sin (2\pi t/\tau + \pi/4)$.

Here λ = the thermal conductivity of the soil and C = the heat capacity of unit volume of soil.

3 THE DATA OF BRIGGS AND SHANTZ IN COMPARISON WITH THE THEORETICAL VALUES OF EVAPORATION

BRIGGS and SHANTZ measured the transpiration by weighing two sealed pots with alfalfa plants (*Medicago sativa L.*). The evaporation of porous cup evaporimeters with cups of different size and colour was determined simultaneously. Other evaporimeters in use were a shallow cylindrical tank, a deep tank sunk in the ground and a filter paper evaporimeter.

The shallow tank had a diameter of 91 cm and a depth of 2.5 cm; it was blackened inside and contained a layer of water of 1 cm depth. This tank was mounted about 1 m over level ground on an automatic balance. The filter paper evaporimeter had a filter paper with a diameter of 12.5 cm. This filter paper was supported by a brass disk with a rim of 1 mm height. It was kept wet by communication with a constant-level reservoir through a tube with a small diameter.

The following meteorological quantities were recorded: air temperature, wet-bulb depression, wind velocity and short-wave radiation intensity. The experiments were carried out at Akron, Colorado, in July 1916 during three days of hot dry weather.

BRIGGS and SHANTZ found that the correlation between the hourly values of the transpiration and the corresponding readings of the different evaporimeters was best for the shallow tank, while the filter paper evaporimeter was next best. In determining these correlations only the variabilities of the ratios of the transpiration rate and the evaporation rates were considered; absolute values were not taken into consideration.

For a comparison of the transpiration with the theoretical values of evaporation according to PENMAN's theory we chose the data for July 22, because here the transpiration curve showed a characteristic drop around noon. Most of the quantities in equation (4) could be determined directly from the original data. In order to compute the outgoing long-wave radiation we used PENMAN's empirical formula. The quantity representing the relative duration of sunshine in this formula was estimated by comparing the short-wave radiation curve of July 22 with the corresponding curves for the previous clear days. The wind velocity at 2 m height was found by multiplying the measured value at 1 m height with a constant factor of magnitude 1.1 [2]).

Besides $\underline{E}'_0$ a value $\underline{E}''_0$ was computed with a correction for the heat flux into the soil. Because no data on the thermal properties of the soil were available we took the following plausible values for a moist soil (cf. DE VRIES (1952)): $\lambda = 4.10^{-3}$ cal/cm sec °C $= 14.4$ cal/cm h °C, $C \doteq 0.5$ cal/cm^3 °C. Since $A \approx 8.0$ °C we obtain with (7):

$$S = 11 \sin (2\pi t/24 + \pi/4) \text{ cal/cm}^2 \text{ h}.$$

The results are represented in fig. 1 which is almost self-explanatory. Smooth curves were drawn through the two-hourly values except for those of the wind velocity. The transpiration and evaporation rates are expressed in mm/h. There was some difficulty in the estimation of the effective evaporating area for the transpiration curve. In a previous paper BRIGGS and SHANTZ (1916) gave some values for the area of the shadow cast by the plants on the ground. In the calculation of the transpiration rate the average value of these areas was taken. For comparison the wind velocity and the short-wave radiation curves are included. The latter was expressed in mm/h by dividing the radiation intensity (in cal/cm^2 h) by 0.1 L.

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[2]) This factor holds for a logarithmic wind profile with a roughness parameter $\underline{z_0} =$ 0.5 cm (cf. SUTTON (1949)).

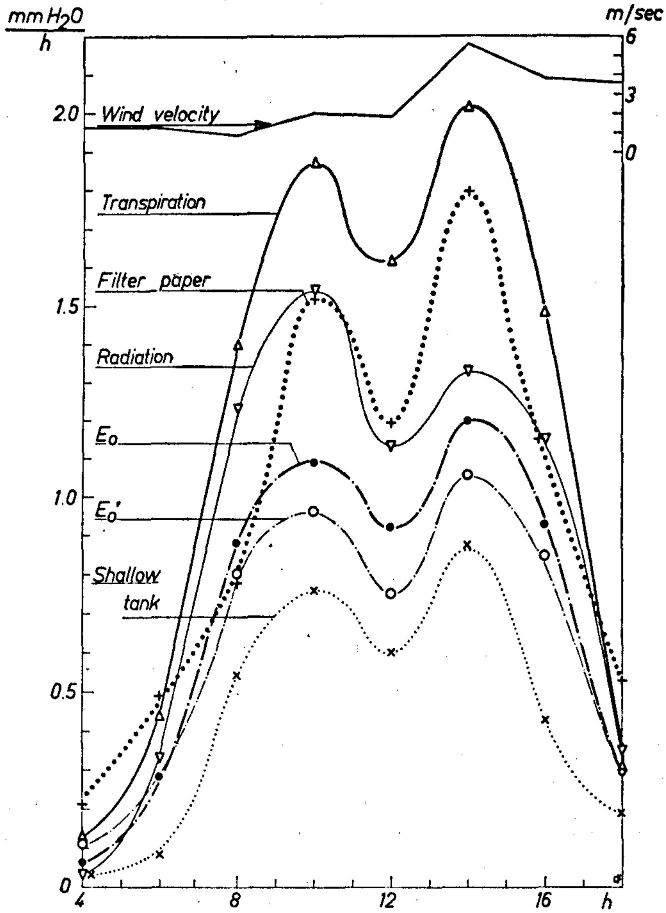

FIG. 1. DAILY VARIATION OF TRANSPIRATION RATE, EVAPORATION RATES, CALCULATED EVAPORATION RATES ($\underline{E}_0$, $\underline{E}'_0$), SHORT-WAVE RADIATION INTENSITY AND WIND VELOCITY FOR AKRON, JULY 22, 1916.

Fig. 2 contains the same results in a somewhat different representation. Here the ratios of the two-hourly values of transpiration and the corresponding values of evaporation or radiation are plotted.

4 Discussion

The predominant feature in fig. 1 is the large influence of the radiation on the transpiration and evaporation, which is for instance clearly demonstrated by the drop in the curves. This, of course, is a consequence of the fact that for the evaporation of 1 mm water a heat-amount of about 58 cal/cm^2 is needed.

The influence of wind velocity is only a secondary one. It is, for instance, reflected in the height of the maximum at 14 h, which is greater than that of the maximum at 10 h for the transpiration and evaporation curves, while for the radiation curve the reverse holds true. This difference is due to the maximum in wind velocity at 14 h.

As a matter of course, the same facts follow from a consideration of the theoretical equation (4). Since the outgoing long-wave radiation was, on the

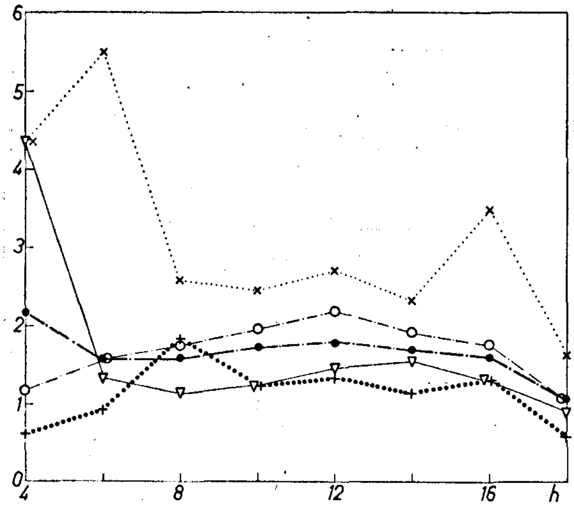

FIG. 2. TRANSPIRATION RATE DIVIDED BY EVAPORATION RATES AND BY RADIATION INTENSITY. The signs correspond to those of the lower curves in fig. 1.

average, about 15 % of the incoming short-wave radiation between 6 and 18 h, H is mainly determined by the latter quantity.

Further it follows from equation (4) that the part of E_0 directly due to radiation equals $\frac{\Delta}{\Delta + \gamma} H$ and the part due to the wind equals $\frac{\gamma}{\Delta + \gamma} E_a$. Now γ is almost independent of temperature ($\gamma = 0.49$ mmHg/°C), whereas Δ increases rapidly with increasing temperature. In our case for instance Δ varied from 1.04 mmHg/°C at 4 h ($T_a = 18.3$ °C) to 2.12 mmHg/°C at 12 h ($T_a = 33.3$ °C), which gives $\Delta/(\Delta + \gamma) = 0.68$ and 0.81 respectively. Therefore it can be seen that the transpiration and evaporation processes are mainly governed by the incoming radiation.

For the maxima at 10 h and 14 h the values of $\frac{\Delta}{\Delta + \gamma} H$ were 0.92 mm/h and 0.85 mm/h, while the corresponding values of $\frac{\gamma}{\Delta + \gamma} E_a$ were 0.17 mm/h and 0.35 mm/h.

Since the factor $\Delta/(\Delta + \gamma)$ rises with the temperature, the influence of the wind velocity on the evaporation will be increasing with falling temperature if $\varepsilon_a - e_a$ remains constant; e.g. at 0 °C the value of Δ becomes 0.36 mmHg/°C. In general, however, the value of $\varepsilon_a - e_a$ will also decrease with temperature, since ε_a falls off rapidly while the relative humidity usually tends to increase. In the absence of incoming radiation H becomes negative and the evaporation will usually be small or dew formation will occur ($E < 0$).

A second point that deserves attention is the difference in magnitude between the various curves in fig. 1. The measured transpiration is far greater than the computed value E_0, hence $E_T > E_0$ in the period under consideration. This difference arises from the fact that PENMAN's considerations refer to a closed homogeneous vegetation with a large horizontal extent. In the case of single plants that project above their surroundings the conditions for heat and vapour exchange between the plants and the air are far more favourable than in a closed vegetation, as PENMAN (1948) has already pointed out (p. 142).

Similar arguments hold in the case where the transpiration is determined by the weighing of cut off parts of plants or trees, even if physiological influences are left out of consideration. These measurements must therefore be interpreted with great care; they will seldom give a representative value of the transpiration from a vegetation as a whole.

The magnitude of the filter paper evaporimeter reading may also be ascribed to the fact that this evaporimeter was exposed at 1 m height over level ground. The extra energy needed for evaporation must have been supplied by the air flowing past the evaporimeter. The evaporation from the shallow tank, on the other hand, is much smaller. The most plausible explanation we could think of is that the rim of this tank, which protrudes 1.5 cm above the water surface, causes the formation of a relatively deep layer of still air over the surface [3]).

The difference in magnitude between the curves $\underline{E}_0$ and $\underline{E}_0'$ is rather small. It must be understood that the estimated value of $\overline{S}$ holds for the case of a moist field soil and does not refer to the conditions in the pot experiments of BRIGGS and SHANTZ.

From fig. 2 it becomes clear that the variability of the ratio between transpiration and calculated evaporation is even less than for the "best" evaporimeters. As a matter of course, the advantage of an evaporimeter lies in its simplicity of construction, maintenance and reading, which also results in a low cost. In most circumstances it provides a picture of the changes in evaporation rate, a quantity that is determined by meteorological conditions. The absolute magnitude of the reading, however, will depend on the construction and exposure of the evaporimeter. In this connection the large reflection of short-wave radiation by a white coloured evaporimeter is of importance. This point was also investigated by BRIGGS and SHANTZ, who found that the difference in reading between a brown and a white coloured porous cup evaporimeter could be ascribed to the enlarged absorption of radiation by the dark coloured cup.

During the night hours the transpiration is usually negligible, whereas the evaporation from an evaporimeter is relatively large, especially at high wind speeds. This is due to the fact that the evaporimeter continuously cools the air flowing past its evaporating surface, in the same way as does a wet bulb thermometer.

The correlation between the hourly values of transpiration and the evaporation from the deep tank was very poor. This is due to the fact that for the deep tank (depth = 60 cm) S in equation (1) is relatively large as a consequence of the penetration of short-wave radiation in the water on one side and of an increased heat transfer by convection in the water on the other side.

Although PENMAN's theory describes the main features of the evaporation and the transpiration process with an accuracy that will be sufficient for many practical purposes we wish to conclude this section by mentioning a few limitations to the theoretical treatment given above. A consideration of the prob-

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[3]) In comparing the shallow tank evaporation with E_0 it must also be remembered that with the former heat exchange with the air takes place at both faces, while evaporation only occurs at the upper side. This argument also holds for the filter paper evaporimeter, however, although there is some difference between both cases, because with the filter paper the absorption will mainly occur at the surface, whereas in the tank this absorption will mainly take place at the blackened bottom.

lems put forward in the following paragraphs may provide a starting point for further development of the theory and may lead to a better insight in the true value of E_T / E_0.

The limitations in question are mainly twofold:

a) There exists a serious doubt about the precise value of the factor f in equation (2). As was already stated f depends on the aerodynamic properties of the earth surface (usually expressed by the so-called roughness height), on the conditions of the thermal stability and on the wind velocities. The latter quantity ultimately depends on the horizontal pressure gradient if the first two quantities have predetermined values.

The thermal stability conditions show a diurnal variation, a point that must be remembered in computing the diurnal variation of evaporation. Recently LETTAU (1949) made an attempt to incorporate the stability conditions in the theory of atmosferic turbulence and we refer to his article for a thorough discussion of this problem.

PENMAN's empirical form for f given in equation (5) was based mainly on experiments with a relatively small water surface. It gives an average value for a period of one or more days, thus averaging over different stability conditions. Although good results have been obtained by using (5) in the computation of E_T, care must be taken in applying this formula to surfaces with different aerodynamic properties. In a recent paper PENMAN (1952) gives a modified form of equation (5) for the case of an orchard.

b) In applying the energy balance concept to a surface carrying a vegetation it must be remembered that the absorption and emission of radiation and the exchange of heat and water vapour between plants and air take place in a layer with a certain vertical extension and not at a geometrical surface as in the case of level ground. So far – to our knowledge – there exists no theory that describes the heat and vapour economics in such a layer.

ACKNOWLEDGMENT

One of us (DE V.) is indebted to Dr H. L. PENMAN for his kindness in showing to him the manuscript of the article "The physical basis of irrigation control" before its appearance in print.

LITERATURE

BRIGGS, L. J. and H. L. SHANTZ: Daily transpiration during the normal growth period and its correlation with the weather. *J. Agr. Res.* 7 (1916) 155.

– –: Comparison of the hourly evaporation rate of atmometers and free water surfaces with the transpiration rate of Medicago sativa. *J. Agr. Res.* 9 (1917) 277.

LETTAU, H.: Isotropic and non-isotropic turbulence in the atmosferic surface layer. *Geophysical Research Papers* No. 1 (1949).

PENMAN, H. L.: Natural evaporation from open water, bare soil and grass. *Proc. Roy. Soc.* A 193 (1948) 120.

– –: The physical basis of irrigation control. Will appear in *Proc. 13th Int. Hortic. Congr.*, London, 1952.

– – and R. K. SCHOFIELD: Some physical aspects of assimilation and transpiration. *Symposia Soc. Exp. Biol.* No. 5 (1951) 115.

RIDER, N. E. and G. D. ROBINSON: A study on the transfer of heat and water vapour above a surface of short grass. *Quart. J. Roy. Met.* Soc. 77 (1951) 375.

SUTTON, O. G.: Atmosferic turbulence. Methuen and Co., London (1949).

VRIES, D. A. DE: Het warmtegeleidingsvermogen van grond. *Mededelingen Landbouwhogeschool, Wageningen* 52 (1952). Verhandeling 1.