POTENTIAL EVAPOTRANSPIRATION 1)

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ABSTRACT

Potential evapotranspiration calculated with Thornthwaite's *empirical formula for several stations is compared with the values following from* Penman's *theory (table 1). Yearly values show a reasonable agreement. Monthly values, however, show a shift in phase, due to the fact that* Penman's *values are nearly in phase with solar radiation, whereas those according to* Thornthwaite *are in phase with temperature. If advective heat transfer is not important, the former view is favoured by theory.*

The amount of water which must be supplied to obtain optimal conditions of soil moisture and vegetation must at least be equal to the water transpired by the plants and evaporated from the soil if the small quantity embodied in the plant is neglected. This minimum water supply under optimal conditions of vegetation is called "Potential Evapotranspiration" (P.E.). The plant seems not to be the regulating factor in transpiration if water is abundantly available. Therefore, P.E. can be calculated as a physical process 2). It is determined by the heat available for evaporation and the transport of water vapour in the lower air layers, the latter factor being a function of wind velocity and of saturation deficit. On this basis, many formulae have been derived by several authors of whom Penman (1948), in particular, has arrived at handy formulae for practical use, by a very elegant elimination of temperature and humidity at the surface of the soil, which in general are not known.

Other methods of calculation of P.E. are based upon empirical correlations with climatological data e.g. the average monthly air temperature. Thornthwaite's (1948) method is at present the most noted representant of this class in Anglo-American literature. His correlation has been compared with a great number of experimental P.E. values. It is based upon the average monthly air temperature and makes use of correction factors for geographic latitude.

By comparing Penman's- and Thornthwaite's methods the present authors have attempted on one hand to gain a better understanding of the meaning of the purely empirical correlation formulae and on the other hand to use the latter ones to improve the physical methods which contain some semi-empirical factors such as e.g. in the formulae for the turbulent heat and vapour transport.

Air temperature at the standard height of 2 metres depends on the temperature of the surface of the soil and on the properties of the air mass itself. The average monthly temperature at the surface is a periodic function of the time which lags behind the average solar radiation owing to the storage of heat in the soil during the summer and its release during the winter. The variation of the monthly temperature is small at low latitudes. At latitudes higher than the tropics its period is one year. The lag in phase of the temperature has been calculated for solar radiation varying with a period of one year.

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2) Cf. Thornthwaite and Holzman (1942), Albrecht (1950), Horney (1950), Pasquill (1950).

At first it has been assumed that evapotranspiration is determined by the prevailing temperature and thus is in phase with the temperature. A shift in phase of 10 days results.

Secondly evapotranspiration has been assumed to be determined by the radiation itself and therefore to be in phase with insolation. This gives a shift in phase of about one month.

In both calculations average values of soil conductivity and volume heat capacity have been adopted and sensible heat has been assumed to be equally divided between soil and air. Average data for actual evapotranspiration for this country have been used i.e. a periodic function of the time with a period of one year and an amplitude of 4.5 cm of water/month.

The calculation proceeds as follows:

Let $T = A \sin \omega t$ be the variable part of the air temperature and $R = K_0 A \sin (\omega t + \varphi)$ the variable part of the insolation. Here ω is the circle frequency $= 2.0 \times 10^{-7}$ sec^{-1} for the yearly variation, A the amplitude of the temperature variation (7° C in the numerical calculation), t the time in seconds, $K_0 A$ the amplitude of the yearly variation of insolation and φ the phase shift between radiation and air temperature. Let further $R_b = BA \sin \omega t$ be the variable part of infrared radiation which is proportional to the air temperature (or strictly to the temperature of the surface of the plants covering the soil). PENMAN's formula (1948) for the calculation of infrared radiation is:

1. $R_b = \sigma T_a^4 (0.56 - 0.09 \sqrt{e_d}) (0.10 + 0.90 \frac{n}{N})$.

In this formula R_b is the infrared radiation, σ STEFAN–BOLTZMANN's constant $= 1.364 \times 10^{-12}$ cal/sec cm^2 °C^4, T_a the absolute temperature of the air which is $284 + A \sin \omega t$ degrees KELVIN. The variable part of T_a^4 equals $4T_a^3 A \sin \omega t$. The average vapour pressure of water $e_d = 7.5$ mm of mercury. n/N is the relative duration of bright sunshine $= 0.50$. Variations of these quantities with time are neglected.

From eq. 1 the variable part of the infrared radiation is calculated as:

$$R_b = BA \sin \omega t = 1.75 \times 10^{-5} A \sin \omega t.$$

If λ is the conductivity of the soil and c the volume heat capacity, than the flow of heat towards the soil at the surface becomes:

$\sqrt{\lambda c \omega}\, A \sin (\omega t + \frac{\pi}{4})$, and it is assumed that an equal amount of heat flows to the air. With a value $\lambda = 4 \times 10^{-3}$ cal/cm sec °C and $c = 0.5$ cal/cm^3 °C one has:

$$\sqrt{\lambda c \omega} = 2.06 \times 10^{-5} \text{ cal/cm}^2 \text{ sec °C}.$$

For the Netherlands the variable part of the heat consumed for evapotranspiration is a periodic function with an amplitude of 4.5 cm of water/month, which is equivalent to an amplitude of 1.0×10^{-3} cal/cm^2 sec. Thus the variable part of energy consumed for evapotranspiration V can be represented by:

2. $V = K_1 A \sin \omega t$ with $K_1 = 0.14 \times 10^{-3}$ cal/ cm^2 sec °C if V is in phase with the temperature, and
3. $V = K_1 A \sin (\varphi t + \omega)$ if V is in phase with insolation.

The shift in phase can now be easily calculated from an energy balance.

For evapotranspiration in phase with air temperature the energy balance becomes:

$$R \sin(\omega t + \varphi) - BA \sin \omega t = K_1 A \sin \omega t + 2A\sqrt{\lambda c \omega}\, \sin\left(\omega t + \frac{\pi}{4}\right) \tag{4}$$

which must be satisfied for all values of t. After developing $\sin(\omega t + \varphi) = \sin \omega t \cos \varphi + \cos \omega t \sin \varphi$ and $\sin\left(\omega t + \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} (\cos \omega t + \sin \omega t)$ and equating to zero the coefficients of $\cos \omega t$ and of $\sin \omega t$ separately one obtains the set of equations:

$$K_0 \sin \varphi = \sqrt{2 \lambda c \omega} \tag{5a}$$

$$K_0 \cos \varphi = \sqrt{2 \lambda c \omega} + B + K_1 . \tag{5b}$$

from which

$$\operatorname{tg} \varphi = \frac{\sqrt{2 \lambda c \omega}}{(B + K_1) + \sqrt{2 \lambda c \omega}} . \tag{6}$$

Inserting the above values for the constants in the right hand side of eq. 6 leads to $\operatorname{tg} \varphi = 0.154$, $\varphi = 0.16$ radians corresponding to 0.3 month.

If evapotranspiration is in phase with insolation then the energy balance results:

$$R \sin(\omega t + \varphi) - BA \sin \omega t = K_1 A \sin(\omega t + \varphi) + 2A\sqrt{\lambda c \omega}\, \sin\left(\omega t + \frac{\pi}{4}\right) \tag{7}$$

which gives:

$$(K_0 - K_1) \sin \varphi = \sqrt{2 \lambda c \varphi} \tag{8a}$$

$$(K_0 - K_1) \cos \varphi = \sqrt{2 \lambda c \omega} + B \tag{8b}$$

and thus

$$\operatorname{tg} \varphi = \frac{\sqrt{2 \lambda c \omega}}{B + \sqrt{2 \lambda c \omega}} \tag{9}$$

In this case one obtains $\operatorname{tg} \varphi = 0.62$, $\varphi = 0.52$ radians $= 1.2$ month.

The experimental data show that temperature lags about one month behind radiation which is in line with the assumption that incident solar radiation is the main cause of evapotranspiration. This conclusion may be subject to revision if the monthly temperature is largely dependent upon the properties of advective air masses.

Table 1 contains some data about evapotranspiration calculated by Penman's and Thornthwaite's methods respectively. A phase shift of one month does actually occur in several cases and the exceptions can readily be accounted for.

The temperatures at San Francisco and Valentia are highly affected by the westerly winds. In St Paul the relative duration of sunshine for June and July is 0.63 and 0.71 respectively, which results in a somewhat higher radiation intensity for July.

The evaporation of a water surface calculated by PENMAN's method using a reflection coefficient 0.05 is denoted by E_0. The shape of the curve E_0 versus time in many cases is very similar to the curve P.E. versus time except for the shift in phase. A reflection coefficient of about 0.1 in E_0 would give the same absolute yearly value of both. The column E_T contains values calculated with PENMAN's semi-empirical reduction factor applicable to vegetation. The low yearly P.E. at Grand Junction and San Francisco is due to low monthly temperatures owing to the elevation and the cool westerly winds respectively. Experimental determination of P.E. at these or similar stations would provide very valuable information about the relative importance of convective heat supply and moisture transport as compared with radiation.

That radiation is an important factor follows also from THORNTHWAITE's empirical formula. The slope of the curve log t versus log P.E. generally increases with latitude which is a consequence of the fact that at high latitudes a higher radiation intensity occurs at a given temperature than at low latitudes.

Discrepancies of calculated and measured P.E. as e.g. reported by LEEPER (1950) for Australian stations can be easily explained by taking the radiation into account. Sydney with a calculated P.E. for July of 106 mm and a measured tank evaporation of 71 mm is a coastal station, whereas Alice Springs with 71 mm and 142 mm respectively lies in the desert. The average cloudiness at these two stations for July is 0.44 and 0.14 respectively.

It is further interesting to observe that THORNTHWAITE's empirical relationship of P.E. and average monthly temperature which is represented by a straight line on a double logarithmic scale is approximately obtained if E_0 is plotted against the average monthly temperature. Actually two lines are obtained in the latter case one for increasing the other for decreasing temperatures which is again a consequence of the phase shift in temperature. In the first part of the growing season a certain radiation intensity corresponds to a temperature which is lower than that in the second part. The empirical formula corresponds more to the temperature in the second part of the growing season. Apparently more weight is carried by the later stages of the growing season when the vegetation covers the soil more completely and the plants are taller.

Table 1. Comparison of evaporation data in cm/year calculated according to THORNTHWAITE (P.E.) and PENMAN (E_0, E_T).

Station	Coördinates	Elevation (m)	P.E.	E_0	E_T	Phase shift
Djakarta	6°11′ S, 106°50′ E	8	460	460	345	irregular
Galveston	29°18′ N, 94°50′ W	16	112	166	124	1 month
Raleigh [1]	35°45′ N, 78°37′ W	115	89	110	83	„
San Francisco	37°48′ N, 122°26′ W	47	73	113	85	2 weeks
Grand Junction	39° N, 108°34′ W	1403	74	184	138	1 month
St Paul	44°58′ N, 93° 3′ W	255	64	92	69	0
Valentia	51°56′ N, 10°15′ W	9	64	68	51	2 weeks
De Bilt	52° 6′ N, 5°11′ E	2	67	77	58	1 month
Trondheim	63°26′ N, 10°25′ E	40	51	60	45	1 month

[1] Cf. VAN BAVEL and WILSON (1952).

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THE EFFECT OF A IODINATED CASEIN FRACTION ON THE MILK YIELD OF GOATS [1]

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SUMMARY

MØLLGAARD has advanced the idea that iodinated casein might contain two hormones, one being thyroxine, stimulating metabolism, and the other a factor only effecting an increase in milk yield. Experiments made by POULSEN with lactating goats seemed to confirm this view. Our experiments do not agree with these ideas. The effect of iodinated casein and the so-called thyroxine-free lactogenic factor (fraction B), isolated from hydrolysed iodinated casein, was investigated by experiments on a number of lactating goats. The effect on milk yields was favourable though somewhat variable. Also an increase of the butterfat yield was clearly noticeable where fraction B was administered. The increased milk yields declined rapidly as soon as the injections were discontinued. Live weight diminished during the experiments by about 10 percent on an average and heartrate and body temperature were noticeably increased.

The results obtained with fraction B were in no way more favourable than those obtained with iodinated casein.

1 INTRODUCTION

Amongst the hormones applied experimentally in livestock nutrition and in some countries also in dairy farming, the thyroid hormone thyroxine is to be mentioned in the first place. Administration to lactating animals usually takes place as iodinated casein and in doing so it can exert a very favourable influence on the milk yield as well as on the butterfat yield. A temporary increase of 20 percent and over in milk yield is no exception.

By administering iodinated casein, viz. thyroxine, metabolism is intensified, the increase of the milk and butterfat yield being one of the symptoms, as far as not inhibited by other factors. Other, less desirable consequences of the administration of iodinated casein are: an increased heartrate and higher body temperature, loss of weight and possibly some other symptoms indicative of hyperthyroidism. Besides, after the administration of iodinated casein is dis-

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