

## **A modified Mitscherlich equation for rainfed crop production in semi-arid areas: 2. Case study of cereals in Syria**

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### **Abstract**

In a companion paper, a theoretical framework for the application of the Mitscherlich equation to describe nutrient response for rainfed crop production is presented. Water-limited potential, or maximum, yield is assumed to be a linearly increasing function of seasonal rainfall. In addition, the quantity of nutrients required by a crop to achieve water-limited potential yield and the nutrient availability are also expressed as functions of seasonal rainfall. This rainfall-dependent Mitscherlich equation is evaluated using data from agronomic (nitrogen  $\times$  phosphorus) experiments conducted under rainfed conditions in the semi-arid region of Syria. The analysis showed that rainfall explained most of the variation in yield between locations and seasons, but that the moisture-dependent nutrient part of the Mitscherlich equation explained a small but significant part of the yield variation as well. The model discussed in this paper appears to provide a framework for a Mitscherlich-type approach to describing crop response to nutrient availability c.q. fertilizer application under rainfed conditions in the semi-arid regions.

**Keywords:** nutrient-use efficiency, nutrient availability, nitrogen, phosphorus, nutrient uptake, water balance

### **Introduction**

Maximum achievable, or potential, yields under rainfed conditions in the semi-arid regions of the world are ultimately limited by available moisture up to a certain threshold yield level where other factors, such as photoperiod, carbon dioxide level, temperature or radiant energy may become limiting. Biomass production is known to be zero at zero rainfall and then increases with increasing rainfall, above a certain non-zero threshold level. Therefore, water-limited potential yield of a particular crop can be described, in first approximation, by a linear function of rainfall, for a limited range of agro-ecological (rainfall) and edaphic conditions.

Furthermore, nutrient availability and nutrient-use efficiency are also affected by available moisture in the soil. For example, if there is no moisture in a certain layer

of the soil (e.g., the top layer) then the nutrients contained in that layer are inaccessible to the crop. Similarly, soil phosphorus is known to be less available to crops under dry conditions than at higher levels of available moisture (Matar *et al.*, 1992; Harmsen, 1995). At the other end of the spectrum, under high rainfall conditions, leaching of nitrate to beyond the reach of the rooting system of the crop, or denitrification could effectively decrease the availability of soil mineral nitrogen.

In the classical Mitscherlich equation, the maximum, or potential, yield is assumed to be a constant (Mitscherlich, 1913). A modification of the classical Mitscherlich equation (Harmsen, 2000) considers the effects of moisture on water-limited potential yield, nutrient uptake and nutrient availability in a simple and direct way. This model could provide a framework for a Mitscherlich-type approach to describing crop response to nutrient availability c.q. fertilizer application under rainfed conditions in the semi-arid regions. However, the model needs to be evaluated against experimental data and the assumptions underlying the model need to be tested and discussed. The aim of the present paper is to test the modified Mitscherlich equation (Harmsen, 2000) using data from agronomic (nitrogen  $\times$  phosphorus) experiments conducted in the semi-arid region of Syria.

## Materials and methods

### *Field trials*

To evaluate the modified Mitscherlich equation, use is made of a dataset collected in the early eighties in Syria. During the 1982/83 and 1983/84 growing seasons, variety verification and agronomic trials were conducted jointly by the Agricultural Research Center (ARC) of the Ministry of Agriculture and Agrarian Reform (MAAR) of the Syrian Arab Republic at Douma, near Damascus, and the International Center for Agricultural Research in the Dry Areas (ICARDA) at Tel Hadya, near Aleppo. The agronomic results of a limited number of the variety verification trials have been reported by Anderson (1985a-b). Yield data obtained in these trials and experimental details have been reported in two mimeographed reports, entitled 'Results of the cereals 1982/83 on-farm trials', ARC and ICARDA, Syria, September 1983', and 'Results of the cereal field verification trials 1983-84', ARC and ICARDA, Syria, September 1984. Results from soil and plant analysis have not been reported previously.

The present paper summarizes some of the agronomic results, including the results of soil and plant analysis, of the 1982/83 and 1983/84 N $\times$ P trials, conducted alongside the variety verification and other agronomic trials on the same sites. The emphasis in the present paper is on the use of the dataset for the validation of the modified Mitscherlich equation. The primary objective of the N $\times$ P trials was to test the effect of fertilizer application on cereal yields under a range of soil and climatic conditions. These trials were considered an essential step in producing technologies suitable for increasing cereal productivity at the farm level.

During the 1982/83 and 83/84 growing seasons, 28 trials were conducted at 24 dif-

ferent locations, covering most of the barley- and wheat-producing region of Syria. The sites differed in estimated long-term annual rainfall (Table 1). Two trials were conducted under conditions of supplementary (Hama, 83/84) or full irrigation (Saa-lo, 83/84). These two trials represent conditions where water would not be a constraint to production, and are included for reference. The cropping history of the sites and the fertilizer use in previous seasons, to the extent that this information could be obtained from the farmers, is summarized in Table 2.

### *Experimental treatments*

Treatments in the N×P trials differed between the major agricultural zones that were distinguished by ICARDA in Syria on the basis of estimated mean annual rainfall:

Zone A: >350 mm y<sup>-1</sup>

Zone B: 250–350 mm y<sup>-1</sup>

Zone C: <250 mm y<sup>-1</sup>

During the 1982/83 season, the N×P trials were conducted in farmers fields in a complete factorial design with one replication per site. Fertilizer rates were 0, 30, 60, 90 and 120 kg P<sub>2</sub>O<sub>5</sub> ha<sup>-1</sup> for phosphorus at all sites, and 0, 40, 80, 120 and 160 kg N ha<sup>-1</sup> for nitrogen at sites in zone A, and 0, 30, 60, 90 and 120 kg N ha<sup>-1</sup> at sites in zones B and C. Crop varieties were breadwheat (var. S311 × Norteno) in zone A, durumwheat (var. Sahl) in zone B, and barley (var. ER/Apam) in zone C.

During the 1983/84 season, the N×P trials were conducted in farmers fields in a complete factorial design with two varieties per site, each in one replication. Crop varieties were breadwheat (vars. Bohouth and Sham) under irrigation and in zone A and durumwheat (vars. Sebou and Douma 221) in zone B. The trials in zone C were not harvested, because of crop failure due to drought in this zone. As there were no significant differences between varieties at any of the sites in the N×P trials, the yield data were pooled for each of the sites and the yield data of the two varieties were treated as replicates. Fertilizer rates were 0, 60 and 120 kg P<sub>2</sub>O<sub>5</sub> ha<sup>-1</sup> for phosphorus at the irrigated sites, 0, 40 and 80 kg P<sub>2</sub>O<sub>5</sub> ha<sup>-1</sup> in zones A and B, except for two sites in zone B (Seraqeb and Suran) where the rates were 0, 30 and 60 kg P<sub>2</sub>O<sub>5</sub> ha<sup>-1</sup>. For nitrogen the rates were 0, 40, 80, 120 and 160 kg N ha<sup>-1</sup> at the irrigated sites, 0, 40, 80 and 120 kg N ha<sup>-1</sup> in zone A and 0, 30, 60, 90 kg N ha<sup>-1</sup> in zone B, except for Seraqeb and Suran where the rates were 0, 40 and 80 kg N ha<sup>-1</sup> (see also Table 4).

Land preparation was done by the farmers. Seeding, chemical weed control, harvesting, and soil and crop sampling was done by the staff of ARC and ICARDA. All trials were sown with an Oyjord seed drill, row spacing was 20 cm and the plot size was 1.6 × 10 m. Weeds were controlled in all plots with a post-emergence herbicide (Brominil or Brominil-Plus, and Illoxen for grass weeds). Grain yields at harvest were obtained by harvesting the central six rows in each plot with a Hege combine, that is, leaving one row at each side of the plot.

*Soil sampling procedures*

Soils at all experimental sites were sampled at the start of the growing season, from November to December. Where possible, soils were sampled down to 120 cm depth, at three locations at each site. All samples were stored in freezers prior to processing. Samples were then air-dried, ground to pass a 2-mm sieve and stored. Prior to analysis, soil samples of the same depth, from different replicates at the same site, were mixed, using a riffle-type sample splitter to obtain equal amounts of soil. For each chemical analysis, soil samples were subsampled using a riffle-type sample splitter and finely ground, using an agate or porcelain mortar.

In the N×P agronomy trials, soil samples were also taken in late winter, from the end of January to early March, in fallowed alleyways between experimental fields. At harvest, soils in these trials were sampled in fertilized and unfertilized cereal plots, down to 120 cm (zone C), 150 (zone B) or 180 cm depth (zone A and irrigated trials).

*Analytical procedures*

Analytical procedures are described in the ICARDA Laboratory Manual (Ryan *et al.*, 1996). Soil moisture was determined by drying representative field-moist soil samples to constant weight at 105 °C. Bulk density was estimated from detailed measurements in Aleppo Province and extrapolation on the basis of clay and moisture content of the soils. Mechanical analysis (% w/w) was determined in subsamples of 40 g of air-dried soil (2 mm fraction) by the hydrometer method for the silt plus clay and clay fractions, and by wet sieving (53 mm sieve) for the sand fraction, at a temperature of 20 °C. The hydrometer method was calibrated against the sedimentation method (Matar *et al.*, 1987).

For the titrimetric determination of calcium carbonate (lime) equivalent (% w/w), an air-dried soil sample was ground to pass a 150 mm sieve. A 1 g subsample was treated with a known volume of 1 N HCl. After equilibration, the unused acid was back-titrated with a 1 N NaOH solution using a phenolphthaleine indicator. pH was determined in a 1:1 (w/v) suspension of 50 g of air-dried soil (2 mm fraction) and 50 ml of distilled water. After equilibration for one hour, measurements were taken in a stirring suspension (magnetic stirrer) using a combined electrode; electrode readings were taken after 30 seconds (at 20 °C). For the measurement of the electrical conductivity, EC (mS cm<sup>-1</sup> at 20 °C), a 1:1 (w/v) soil-solution extract was obtained by equilibrating 50 g of air-dried soil (2 mm fraction) and 50 ml of distilled water for one hour, and by filtrating the suspension (using suction). The conductivity cell was immersed in the clear filtrate and measurements were taken after 30 seconds in a standing solution.

Available phosphorus in soil was extracted with a sodium bicarbonate solution (Olsen *et al.*, 1954) and phosphorus in the extract was determined according to Murphy & Riley (1963). Mineral ('available') nitrogen in the soil was extracted with water (nitrate-N) and a potassium chloride solution (ammonium-N) according to Buresh *et al.* (1982) and both forms of nitrogen were determined analytically by

steam distillation (Buresh *et al.*, 1982; Ryan *et al.*, 1996).

Oxidizable organic carbon (% w/w) was determined in a finely-ground (150 mm fraction) subsample, according to Walkley & Black (1934). Total organic carbon was estimated by multiplying the results with a factor of 1.33. Total nitrogen was determined in air-dried soil (150 mm fraction) according to a modified Kjeldahl method, including salicylic acid to include nitrate-nitrogen (Bremner, 1965; Nelson & Sommers, 1972; Ryan *et al.*, 1996).

## Results and discussion

The dataset from Syria is used primarily to discuss how the modified Mitscherlich equation could be tested under field conditions, to illustrate the methodology and to discuss the assumptions underlying the model. The dataset is too limited to estimate all parameters in the modified Mitscherlich equation with sufficient accuracy, or to allow for the extrapolation or generalization of the results to the Mediterranean climatic environment, or even to Syria. Nevertheless, the dataset is thought to be relevant to the discussion of the assumptions underlying the modified Mitscherlich equation and for providing some indications as to the values of the relevant parameters.

### *Rainfall conditions and zonation*

Syria has a Mediterranean-type climate with annual rainfall concentrated during the winter period. The growing season for rainfed cereal crops starts in November-December and lasts till May-June. Hence, rainfall is usually reported for the growing season, rather than for the calendar year. Rainfall data for the experimental sites were obtained from meteorological stations of the Syrian Department of Meteorology closest to the experimental sites. However, in some cases the distance between the recording meteorological station and the experimental site was considerable, such that the rainfall data should be considered as fairly rough estimates of seasonal rainfall. Also, the quality of the measurements might differ between the meteorological stations.

It can be seen from Table 1 that some of the sites in Zone C have an estimated annual rainfall higher than 250 mm. This may be partly due to the fact that the accuracy of the rainfall estimates, in particular in the lower rainfall areas, may not be very high. Another reason may well be that in some cases sites have been wrongly classified as belonging to a particular zone.

From Table 2 it follows that the common two-course rotation in the lower rainfall areas is cereal-fallow, where the cereal crop can be barley or durumwheat. In some cases the rotation is cereal-summer crop, where the summer crop (melon or sesame) is grown on stored soil moisture. In the higher rainfall areas the two-course rotation shifts to cereal-legume, with breadwheat becoming more important as the cereal crop and chickpea being the predominant legume crop. In case a fallow (or two fallows!) preceded the experimental crop in a N×P trial, stored moisture may have been

Table 1. Experimental sites: geographical name, province ('Mohafaza': governate, area under the jurisdiction of a governor), agricultural zone, mean annual rainfall (mm y<sup>-1</sup>), estimated from the Climatic Atlas of Syria (Anonymous, 1977), and season that the N×P trials were conducted. Saalo is irrigated (irr) and Hama had supplementary irrigation (suppl).

Site	Province	Zone	Rainfall	Season
Saalo	Deir Ez-Zor	irr	140	83/84
Hama	Hama	suppl	370	83/84
El Ghab	Hama	A	550	82/83
				83/84
Malkieh	Hassakeh	A	500	83/84
Kawkabeh	Aleppo	A	490	82/83
Hayyaleen	Hama	A	480	82/83
Himo	Hassakeh	A	440	82/83
				83/84
Khan Shekhoun	Idleb	A	370	82/83
Sahm Gholan	Daraa	A	360	83/84
Seraqeb	Idleb	B	370	82/83
				83/84
Sheikh Meskin	Daraa	B	350	82/83
Suran	Hama	B	340	82/83
				83/84
Aybtein	Aleppo	B	320	83/84
Nassrieh	Aleppo	B	310	82/83
Tel Abiad	Raqqqa	B	300	82/83
Ezraa	Daraa	B	290	83/84
Ghzeilan	Raqqqa	B	280	83/84
Um El Assafir	Hassakeh	B	280	82/83
Shinshar	Homs	C	300	82/83
Sfireh	Aleppo	C	290	82/83
Marhamieh	Aleppo	C	280	82/83
Tel Khudar	Hassakeh	C	280	82/83
Homaimeh	Aleppo	C	270	82/83
Bir Hashem	Raqqqa	C	200	82/83

carried over from the previous season to the experimental crop, thus possibly obscuring the correlation between actual rainfall in the experimental year and crop yields. Crop rotation may also affect nutrient availability: e.g., the availability of nitrogen may increase after chickpea, lentil or forage (vicia) crops and the same crops may also affect the availability of soil phosphorus through the release of organic compounds (root exudates) that solubilize and complex inorganic phosphorus in soil. Similarly, nutrient availability may increase after fallow.

From Table 2 it further follows that fertilizer use was rather low, in particular during the year preceding the experimental crop. This reflects that if fertilizer is applied at all, it will often be to the cereal crop and less so to the following legume or summer crop. Furthermore, 9 of the sites were fallowed in the year before the experimental crop. Nevertheless, some residual effects of fertilizers could have been present, in particular of phosphorus at El Ghab, Saalo and Sahm Gholan.

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Table 2. Experimental sites in the 1982/83 and 1983/84 growing seasons: cropping history and previous fertilizer use. Fertilizer use is expressed as kg N ha<sup>-1</sup> (nitrogen) and kg P<sub>2</sub>O<sub>5</sub> ha<sup>-1</sup> (phosphorus).

Site	Previous crop		Fertilizer use			
	1980/81	1981/82	1980/81		1981/82	
			N	P <sub>2</sub> O <sub>5</sub>	N	P <sub>2</sub> O <sub>5</sub>
Kawkabeh	wheat	chickpea	40	60	0	25
Hayyaleen	wheat	melon	50	30	0	0
El Ghab	wheat	sugar beet	80	40	150	150 #
Himo	wheat	forage @	80	60	0	0
Khan Shekhoun	wheat	melon	50	40	0	0
Nassrieh	wheat	melon	40	60	0	0
Seraqeb	wheat	fallow	30	20	0	0
Suran	wheat	melon	40	30	0	0
Sheikh Meskin	tomato	lentil	40	30	\$	\$
Tel Abiad	barley	fallow	20	0	0	0
Um El Assafir	wheat	fallow	0	0	0	0
Homaimeh	barley	fallow	0	0	0	0
Sfireh	fallow	fallow	0	0	0	0
Marhamieh	barley	fallow	0	0	0	0
Shinshar	fallow	melon	0	0	0	0
Bir Hashem	fallow	fallow	0	0	0	0
Tel Khudar	wheat	fallow	25	25	0	0
Salamieh	—	—	—	—	—	—

Site	Previous crop		Fertilizer use			
	1981/82	1982/83	1981/82		1982/83	
			N	P <sub>2</sub> O <sub>5</sub>	N	P <sub>2</sub> O <sub>5</sub>
Hama	wheat	chickpea	80	60	0	0
Saalo	wheat	sugarbeet	150	100	150	100
El Ghab	wheat	chickpea	80	60	0	0
Sahm Gholan	melon	melon	50	40	50	40
Himo	wheat	vicia	80	60	0	0
Malkieh	wheat	melon	120	80	0	0
Aybtein	wheat	melon	25	0	0	0
Seraqeb	wheat	melon	50	30	0	0
Suran	barley	lentil	30	20	0	0
Ezraa	wheat	sesame	0	0	0	0
Ghzeilan	wheat	fallow	0	0	0	0

# 75 kg K<sub>2</sub>O ha<sup>-1</sup> used in 1981/82

@ vicia sativa

\$ animal manure used in 1981/82

*Baseline soil data*

Table 3 shows that all soils are quite heavy-textured, ranging from clay loam and silty clay loam in the lower rainfall areas to silty clay and clay in the higher rainfall areas. All soils contain lime: lime contents tend to decrease with increasing rainfall. The few soils that are significantly lower in lime (e.g., Shinshar and Sahm Gholan) are formed in basaltic materials. As a result of the presence of lime, pH in all soils is in the alkaline range (8.0–8.4). Electrical conductivity is fairly low at all sites, except for Bir Hashem, which is slightly saline. Organic carbon and total nitrogen are low in all soils, except for El Ghab, and most C/N ratios are in the range of 8–10, that is, these soils are carbon-limited. This means that when carbon is added to a soil (e.g., crop residues, root mass), nitrogen may be immobilized upon the decomposition of organic matter, shortly after the onset of the rainy season. Soils in

Table 3. Baseline data of the soils (0–20 cm) at the experimental sites: mechanical analysis (% w/w), lime equivalent (% w/w), electrical conductivity (EC; mS cm<sup>-1</sup>), pH, total organic carbon (TOC; % w/w) and total nitrogen (N-tot; % w/w).

Site	Sea- son	Mechanical analysis			Lime Equiv	EC	pH	TOC	N-tot
		Clay	Silt	Sand					
Saalo	83/84	33.5	37.6	33.3	19.4	0.52	8.4	0.60	–
Hama	83/84	60.9	23.6	12.6	30.7	0.53	8.2	0.89	–
El Ghab	82/83	60.0	36.0	0.9	27.4	0.30	8.4	1.44	0.133
	83/84	66.0	32.7	1.0	25.3	0.25	8.4	1.34	–
Malkieh	83/84	63.9	29.7	2.0	19.9	0.35	8.2	0.70	–
Kawkabeh	82/83	54.5	34.9	8.3	24.5	0.32	8.2	0.89	0.096
Hayyaleen	82/83	54.0	31.1	13.1	12.0	0.23	8.2	0.77	0.085
Himo	82/83	48.5	44.0	5.3	30.3	0.34	8.2	0.55	0.068
	83/84	50.3	42.3	6.6	28.7	0.30	8.2	0.89	–
Khan Shekhoun	82/83	55.0	32.8	10.0	29.3	0.23	8.0	0.59	0.066
Sahm Gholan	83/84	71.1	21.5	3.3	4.2	0.19	8.2	0.45	–
	82/83	49.0	35.0	11.9	31.0	0.21	8.3	0.49	0.058
Seraqeb	83/84	55.9	35.9	16.4	30.7	0.27	8.2	0.69	–
	82/83	55.0	36.6	4.9	14.4	0.26	8.2	0.41	0.047
Sheikh Meskin	82/83	49.0	33.7	14.2	33.9	0.23	8.3	0.56	0.060
	83/84	52.1	29.2	18.0	35.6	0.22	8.2	0.60	–
Aybtein	83/84	42.0	29.5	25.6	41.4	0.23	8.4	0.44	–
Nassrieh	82/83	43.5	40.1	14.6	26.9	0.19	8.4	0.37	0.051
Tel Abiad	82/83	35.5	52.7	7.6	15.4	0.33	8.3	0.98	0.110
	83/84	64.6	27.0	3.7	13.4	0.22	8.3	0.37	–
Ghzeilan	83/84	31.3	44.0	21.7	24.5	0.26	8.3	0.70	–
Um El Assafir	82/83	28.5	52.2	19.3	34.8	0.35	8.3	0.65	0.075
Shinshar	82/83	36.0	31.6	30.0	5.8	0.22	8.3	0.92	0.108
Sfireh	82/83	45.0	39.1	14.0	52.6	0.35	8.5	1.05	0.111
Marhamieh	82/83	30.5	46.9	22.0	32.2	0.30	8.4	0.76	0.089
Tel Khudar	82/83	31.5	47.8	17.6	30.5	0.31	8.4	0.59	0.068
Homaimeh	82/83	43.0	49.0	5.8	23.3	0.35	8.4	0.85	0.103
Bir Hashem	82/83	25.0	48.8	25.9	27.6	2.48	7.9	0.48	0.035



the Ghab valley are slightly higher in organic carbon, because of peat residues in the valley soils, dating back to the time that peat widely occurred in the Ghab valley.

*Modified Mitscherlich equation: maximum yields estimated per nutrient per location*

The dataset in Table 4 was used to test the yield response to nitrogen and phosphorus application, using a modified Mitscherlich equation of the form (Harmsen, 2000):

$$GY = GY_x - GY_x \exp(-(\epsilon_{ns} N_s + \epsilon_{nf} N_f) GY_x^{n-1}) \quad (1)$$

where  $GY$  is grain yield ( $\text{kg grain ha}^{-1} \text{ year}^{-1}$ ),  $GY_x$  is the maximum grain yield ( $\text{kg grain ha}^{-1} \text{ year}^{-1}$ ), determined from a Mitscherlich-type equation for each site and nutrient individually, 'exp' denotes the exponential function,  $N_s$  and  $N_f$  are the quantities of nutrient ( $\text{kg nutrient ha}^{-1}$ ) derived from internal ( $N_s$ ) and external ( $N_f$ ) sources, and where  $\epsilon_{ns}$  and  $\epsilon_{nf}$  are 'activity' coefficients ( $\text{kg}^{1-n} \text{ grain kg}^{-1} \text{ nutrient ha}^n \text{ year}^{n-1}$ ), which are a measure for the availability of nutrients derived from internal sources ( $\epsilon_{ns}$ ) and external sources ( $\epsilon_{nf}$ ). It can be seen that the dimension of  $\epsilon_{ns}$  and  $\epsilon_{nf}$  depends on the power constant 'n': if  $n = 1$  it follows that the dimension of the activity coefficients reduces to  $\text{kg}^{-1} \text{ nutrient ha}$ . The expression  $GY_x^{n-1}$  basically represents the moisture-dependence of the exponential function; however, as the values of  $GY_x$  are determined from individual plot yields, for both N and P, and not from rainfall for all sites combined, the notation  $GY_x$  is used rather than  $GY_0$  (cf. Equations 2 and 3).

In order to test the relative effects of rainfall and phosphorus on yields across sites, first a linear regression of  $GY$  on  $GY_x$  was carried out, to assess the importance of maximum yield levels per site (for P only), in explaining the variation in yield across sites and nutrients. This regression gave a value of  $R^2 = 0.947$  (Table 5). This confirms that the differences in maximum yield levels between sites explain most of the variation in yield and that the yield responses to fertilizer application, though highly significant at a number of sites, contributed less to the yield variation in the entire dataset.

The next step was to test Equation (1) for  $n-1 = 0$  (i.e., no moisture dependence); see Table 5. Finally, optimization of Equation (1) for the power constant resulted in a value of  $n-1 = 0.4$ . Although the difference between  $R^2 = 0.966$  ( $n-1 = 0$ ) and the maximum value obtained,  $R^2 = 0.969$  ( $n-1 = 0.4$ ), was very small, a plot of  $R^2$  versus  $n$  yielded a clear parabolic curve, with a maximum at  $n-1 = 0.4$ .

The dataset in Table 4 was also used to test the yield response to applied nitrogen, using the modified Mitscherlich equation (1), where the values of  $GY_x$  were estimated for nitrogen, for each trial individually. A linear regression of  $GY$  on  $GY_x$  resulted in  $R^2 = 0.886$ , which would suggest that, also in the case of nitrogen, variation in maximum yield between sites is the major factor explaining variation in yield. Optimization of Equation (1) with regard to the power constant increased  $R^2$  from 0.942 ( $n-1 = 0$ ) to 0.975 ( $n-1 = -0.7$ ). Although the difference between  $R^2 = 0.942$  ( $n-1 = 0$ ) and  $R^2 = 0.975$  ( $n-1 = -0.7$ ) is not very large, a plot of  $R^2$  versus  $n$  yielded a clear parabolic curve with a maximum at  $n-1 = -0.7$ .

Table 4. Estimated seasonal rainfall (mm season<sup>-1</sup>), nitrate-nitrogen (kg ha<sup>-1</sup>) and P-Olsen (ppm) of the soil at the start of the growing season, and grain yields (kg ha<sup>-1</sup>) for phosphorus (kg P<sub>2</sub>O<sub>5</sub> ha<sup>-1</sup>) and nitrogen (kg N ha<sup>-1</sup>) for all sites (name, season, zone). Seasonal rainfall for the irrigated sites is set at 600 mm season<sup>-1</sup>. Nitrate-nitrogen refers to a depth of 0–60 cm (< 300), 0–90 cm (300–400) and 0–120 cm (> 400), where actual seasonal rainfall is indicated in parenthesis (mm season<sup>-1</sup>). P-Olsen refers to the 0–20 cm layer. Yield responses to phosphorus are for the P-levels indicated, at the optimum N-level (or mean values of 2 N-levels). Yield responses to nitrogen are for the N levels indicated, at the optimum P-level (or mean values of 2 P-levels).

Site	rain	n0	p0	Phosphorus			Nitrogen		
				nf	pf	gy	pf	nf	gy
Saalo 83/84 IRR	[600.0]	71.4	8.1	150	0.0	7047	52.4	0	4521
					26.2	7175		60	5449
					52.4	7060		120	6625
								180	7495
Hama 83/84 IRR	[600.0]	236.6	24.3	90	0.0	6125	52.4	0	4574
					26.2	6177		60	5428
					52.4	6167		120	5277
								180	5136
El Ghab 82/83 A	576.0	44.1	31.5	140	0.0	4480	26.2	0	2350
					13.1	4605		40	3565
					26.2	5034		80	4248
					39.3	5354		120	4922
					52.4	5167		160	4933
El Ghab 83/84 A	512.4	85.5	17.8	100	0.0	6646	34.9	0	4209
					17.5	6193		40	5834
					34.9	6802		80	6703
								120	6902
Malkieh 83/84 A	370.1	70.6	6.8	100	0.0	4803	34.9	0	3308
					17.5	5021		40	4219
					34.9	5021		80	4849
								120	5194
Kawkabeh 82/83 A	307.5	67.1	6.2	140	0.0	2083	45.9	0	2229
					13.1	2042		40	2188
					26.2	2355		80	2104
					39.3	2250		120	2198
					52.4	2209		160	2250
Hayyaleen 82/83 A	441.1	51.3	3.5	140	0.0	2708	45.9	0	2250
					13.1	4313		40	4125
					26.2	4979		80	4062
					39.3	4917		120	4771
					52.4	4000		160	4146
Himo 82/83 A	323.6	49.3	4.3	100	0.0	2334	45.9	0	1980
					13.1	2604		40	2771
					26.2	2584		80	2468
					39.3	2468		120	2459
					52.4	2459		160	2323

# CASE STUDY OF A MODIFIED MITSCHERLICH EQUATION

Table 4. continued

Site	rain	n0	p0	Phosphorus			Nitrogen		
				nf	pf	gy	pf	nf	gy
Himo	206.2	56.8	16.8	20	0.0	329	34.9	0	219
83/84					17.5	339		40	243
A					34.9	358		80	122
								120	153
Khan Shekhoun	350.6	46.1	3.2	140	0.0	4264	45.9	0	2646
82/83					13.1	3792		40	3750
A					26.2	3594		80	3948
					39.3	4396		120	4459
					52.4	3584		160	3521
Sahm Gholan	354.8	53.1	1.5	100	0.0	2211	34.9	0	2390
83/84					17.5	2673		40	2767
A					34.9	3057		80	3034
								120	3080
Seraqeb	362.5	81.4	9.3	105	0.0	4188	45.9	0	3146
82/83					13.1	4417		30	3979
B					26.2	3563		60	3792
					39.3	4271		90	4708
					52.4	4750		120	4313
Seraqeb	220.9	65.4	7.4	60	0.0	3734	26.2	0	2865
83/84					13.1	3688		40	3407
B					26.2	3506		80	3605
Sheikh Meskin	408.1	392.3	7.8	15	0.0	4125	45.9	0	5031
82/83					13.1	3875		30	4511
B					26.2	4448		60	4271
					39.3	4709		90	4386
					52.4	4833		120	5084
Suran	353.0	40.3	3.8	105	0.0	1573	45.9	0	1500
82/83					13.1	1959		30	1686
B					26.2	1886		60	1848
					39.3	1855		90	2230
					52.4	2938		120	2563
Suran	298.8	23.9	3.7	80	0.0	1948	26.2	0	1407
83/84					13.1	2227		40	1682
B					26.2	2530		80	2530
Aybtein	207.4	39.2	3.4	75	0.0	1226	34.9	0	1016
83/84					17.5	1222		30	1313
B					34.9	1156		60	1142
								90	1169
Nassrich	314.0	25.0	3.4	105	0.0	1625	45.9	0	1719
82/83					13.1	1982		30	2250
B					26.2	2067		60	2271
					39.3	2250		90	2396
					52.4	2480		120	2334

Table 4. continued

Site	rain	n0	p0	Phosphorus			Nitrogen		
				nf	pf	gy	pf	nf	gy
Tel Abiad 82/83 B	235.5	40.5	6.1	75	0.0	698	45.9	0	618
					13.1	844		30	875
					26.2	761		60	1083
					39.3	1109		90	1005
					52.4	979		120	1031
Ezraa 83/84 B	299.8	34.9	2.1	75	0.0	1732	34.9	0	1250
					17.5	1856		30	1611
					34.9	1774		60	1576
								90	1972
Ghzeilan 83/84 B	133.1	28.7	3.0	15	0.0	776	34.9	0	875
					17.5	971		30	876
					34.9	971		60	907
								90	755
Um El Assafir 82/83 B	257.3	34.9	2.6	60	0.0	580	45.9	0	896
					13.1	592		30	855
					26.2	754		60	738
					39.3	734		90	646
					52.4	860		120	850
Shinshar 82/83 C	255.3	27.0	4.3	75	0.0	417	45.9	0	292
					13.1	688		30	688
					26.2	813		60	938
					39.3	896		90	1229
					52.4	1271		120	1042
Sfireh 82/83 C	260.9	50.6	7.1	105	0.0	1167	45.9	0	969
					13.1	1240		30	1271
					26.2	1188		60	1042
					39.3	1355		90	1355
					52.4	1292		120	1292
Marhamieh 82/83 C	307.2	39.1	2.9	75	0.0	583	45.9	0	708
					13.1	985		30	938
					26.2	1125		60	1375
					39.3	1500		90	1438
					52.4	1313		120	1250
Tel Khudar 82/83 C	250.4	38.6	2.9	105	0.0	792	45.9	0	1875
					13.1	1370		30	1750
					26.2	1604		60	2021
					39.3	1823		90	1886
					52.4	1855		120	1792
Homaimieh 82/83 C	260.9	38.2	5.1	75	0.0	1417	45.9	0	1479
					13.1	1802		30	2073
					26.2	1927		60	2229
					39.3	2031		90	2198
					52.4	2396		120	2094

# CASE STUDY OF A MODIFIED MITSCHERLICH EQUATION

Table 4. continued

Site	rain	n0	p0	Phosphorus			Nitrogen		
				nf	pf	gy	pf	nf	gy
Bir Hashem	222.6	27.2	3.8	75	0.0	479	45.9	0	563
82/83					13.1	1375		30	2375
C					26.2	1340		60	2081
					39.3	1960		90	2396
					52.4	2517		120	2313

It may be noted that values of the power constant of  $n-1 = 0.4$  for phosphorus and  $n-1 = -0.7$  for nitrogen would indicate that the resulting production functions would be close to the Liebig-model ( $n-1 = -1$ ) in the case of nitrogen and closer to the Mitscherlich-Liebscher model ( $n-1 = 0$ ) in the case of phosphorus (De Wit, 1992, 1994; Harmsen, 2000).

The results obtained so far, are significant in that they show that Equation (1) describes the response to N and P very well, with values of  $R^2$  in the range of 0.969 to 0.975, with one set of values of  $\epsilon_{ns}$ ,  $\epsilon_{nf}$  and  $n$  for all sites, for each of the nutrients. However, the parameter  $GY_x$  relates to the response to either phosphorus or nitrogen at individual sites only, and thus the Mitscherlich equation in the form of Equation (1) cannot be generally applied unless the value of the parameter  $GY_x$  is known. It would thus be attractive if  $GY_x$  could be expressed as a function of rainfall, in which case the equation would apply to either nutrient at all sites, provided rainfall is known.

Table 5. Results of regression analysis for  $GY$ .

Regression equation	Nutrient	$n-1$	$\epsilon_{ns}$	$\epsilon_{nf}$	$R^2$
$a + bGY_x$	Phosphorus	—	—	—	.947
$GY_x - GY_x \exp(-(\epsilon_{ns}N_s + \epsilon_{nf}N_f)GY_x^{n-1})$		0	.336	.0269	.966
		0.4	.0127	.00115	.969
$a + bGY_x$	Nitrogen	—	—	—	.886
$GY_x - GY_x \exp(-(\epsilon_{ns}N_s + \epsilon_{nf}N_f)GY_x^{n-1})$		0	.0142	1.188	.942
		-0.7	5.763	.955	.975
$a + bGY_\theta$	Phosphorus	—	—	—	.743
$GY_\theta - GY_\theta \exp(-(\epsilon_{ns}N_s + \epsilon_{nf}N_f)GY_\theta^{n-1})$		0	.382	.0191	.762
		0.9	.000269	.0000142	.772
$a + bGY_\theta$	Nitrogen	—	—	—	.662
$GY_\theta - GY_\theta \exp(-(\epsilon_{ns}N_s + \epsilon_{nf}N_f)GY_\theta^{n-1})$		0	.0192	.0116	.745
		-0.4	.525	.323	.756

*Modified Mitscherlich equation: water-limited potential yield as a function of rainfall*

Biomass production only occurs when annual rainfall,  $r$  ( $\text{mm year}^{-1}$ ), exceeds a certain threshold value,  $r_0$  ( $\text{mm year}^{-1}$ ), below which no biomass is produced (Harmsen *et al.*, 1983; De Wit, 1994). The relation between grain yield and annual rainfall can thus be approximated as follows (Harmsen, 2000):

$$GY_\theta = GY_p \epsilon_{\theta r} \Delta r \quad (2)$$

where:

$$\Delta r = (r - r_0) \quad (3)$$

and where  $GY_\theta$  is water-limited potential grain yield ( $\text{kg grain ha}^{-1} \text{ year}^{-1}$ ), expressed as a function of rainfall,  $GY_p$  is potential grain yield ( $\text{kg grain ha}^{-1} \text{ year}^{-1}$ ), determined by environmental factors other than water, and  $\epsilon_{\theta r}$  is an activity coefficient which is a measure for the availability of rainfall to the crop ( $\text{mm}^{-1} \text{ year}$ ). Of course, Equation (2) will only be valid for a limited range of rainfall levels, that is, grain yield may increase linearly with rainfall over a limited range, but at some level the relationship would break down because beyond that level rainfall will be no longer limiting yield.

Data from variety verification trials (cf. Anderson, 1985a-b) confirm the assumption of a relation between highest plot yields and rainfall. Taking the highest yielding variety (mean of 3 replicates) of each breadwheat and durumwheat at each site (see Figure 1), the relation obtained for the pooled data of the two seasons was:

$$GY_{\text{plot}} = 13.2(r - 115)$$

with  $R^2 = 0.733$  ( $n = 65$ ) and where  $GY_{\text{plot}}$  stands for the highest 'plot' yields for each of the trials. Hence, the regression was highly significant. At  $600 \text{ mm y}^{-1}$  the highest plot yield would be about  $6372 \text{ kg grain ha}^{-1} \text{ year}^{-1}$ .

As the  $N \times P$  trials were conducted in farmers' fields, land and crop management may not have been optimal. Also, fertilizer application rates may not have been optimal in a number of cases. Hence, the highest yields in these trials may still be somewhat below the 'potential' yields, that is, the yields that would have been achieved under optimal management conditions with only available water limiting yields. Nevertheless, the correlation with rainfall was clear and in line with other observations in the same climatic environment (e.g., Harmsen *et al.*, 1983; French & Schultz, 1984a-b).

A regression of the entire set of  $GY_x$  values estimated for each of the  $N \times P$  trials individually (cf. Table 4), for both N and P, on seasonal rainfall ( $n = 56$ ) resulted in:

$$GY_\theta = 14.0(r - 112)$$

with  $R^2 = 0.734$  and where  $GY_\theta$  is now a function of rainfall only. Hence, at  $600 \text{ mm}$

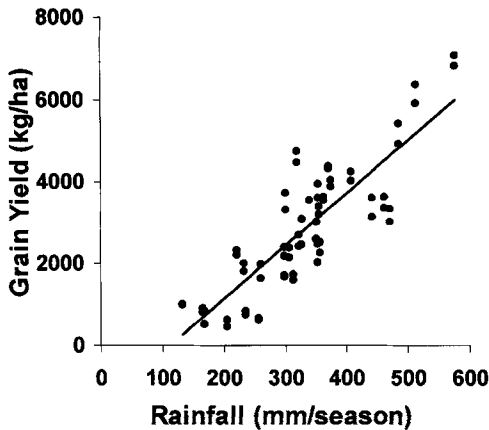


Figure 1. Yields of the highest yielding breadwheat and durumwheat varieties (means of 3 replicates) in variety verification trials conducted during the 1982/83 and 1983/84 seasons in farmers' fields in the semi-arid region of Syria. The solid line represents the linear regression equation:  $GY = 13.2(r-115)$  with  $R^2 = 0.733$  ( $n = 65$ ). Yields ( $\text{kg grain ha}^{-1}$ ) are plotted against seasonal rainfall ( $\text{mm season}^{-1}$ ).

the yield would be about  $6832 \text{ kg ha}^{-1} \text{ year}^{-1}$ . It may be concluded that Equation (2) applies to the dataset in Table 4: maximum yields of cereal trials in the rainfed cereal production region of Syria, estimated from the Mitscherlich equation, tended to increase linearly with increasing seasonal rainfall, in the range of 100–600  $\text{mm year}^{-1}$ .

In related studies in the Mediterranean environment of Northwest Syria, Equation (2) has been found to apply to cereal crops as well as lentil, in the range of about 200–500 mm of seasonal rainfall, and for relatively deep soils on flat land, where leaching losses and surface runoff were limited (Harmsen *et al.*, 1983; Harmsen, 1995). Furthermore, environmental conditions, such as photoperiod, temperature and radiant energy, should be similar for the agro-ecological region considered.

Replacing  $GY_x$  by  $GY_\theta$  in Equation (1) it follows that:

$$GY = GY_\theta - GY_\theta \exp(-(\epsilon_{ns}N_s + \epsilon_{nf}N_f)GY_\theta^{n-1}) \quad (4)$$

where it is thus assumed that water-limited potential yield can be expressed as a function of rainfall only (cf. Equations 2–3).

In the case of phosphorus, a linear regression of  $GY$  on  $GY_\theta$  (cf. Equation 2) resulted in  $R^2 = 0.743$  (cf. Table 5), that is, differences in maximum grain yield, or rainfall, between sites explained 74.3% of the total variation in yield. Optimizing Equation (4) for the power constant 'n', increased the coefficient of variation from 0.762 ( $n-1 = 0$ ) to 0.772 ( $n-1 = 0.9$ ); see Table 5. Hence, the total regression improved by about 3% over rainfall alone. However, a plot of  $R^2$  versus  $n$  showed a clear parabolic curve with a maximum at  $n-1 = 0.9$  (Figure 2).

In the case of nitrogen, a linear regression of  $GY$  on  $GY_\theta$  resulted in  $R^2 = 0.662$ , that is, rainfall alone explained 66.2% of the variation in yield. This result improved by about 8% by using Equation (4) for  $n-1 = 0$  ( $R^2 = 0.745$ ), whereas the value of the power constant that gave the best fit was  $n-1 = -0.4$  ( $R^2 = 0.756$ ); see Table 5 and Figure 2.

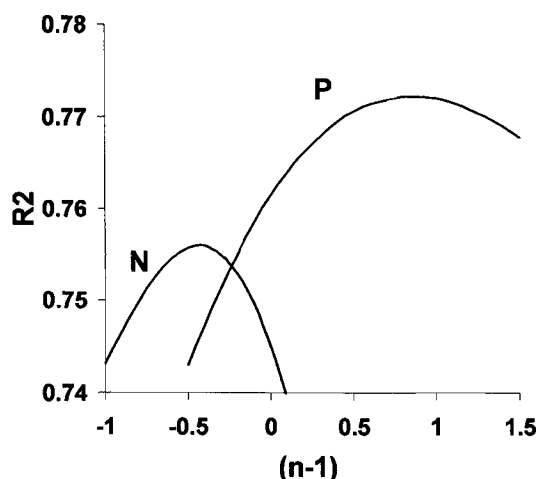


Figure 2. The coefficient of variation ( $R^2$ ) for the regression analysis of the modified Mitscherlich equation (Equation 4) for nitrogen and phosphorus, plotted as a function of the power constant  $n-1$ .

#### *Relation between nutrient availability and rainfall*

The activity coefficients in the modified Mitscherlich equation can be approximated by (Harmsen, 2000):

$$\varepsilon_{s,\theta} = \varepsilon_{ns} GY_{\theta}^n \quad (5)$$

where  $\varepsilon_{s,\theta}$  is a water-dependent activity coefficient ( $\text{kg grain kg}^{-1}$  nutrient) associated with  $N_s$ ; cf. Equation (1). The activity coefficient  $\varepsilon_{f,\theta}$ , associated with  $N_f$ , is defined in a similar manner. As the dimension of  $\varepsilon_{s,\theta}$  is  $\text{kg grain kg}^{-1}$  nutrient, it follows that the dimension of  $\varepsilon_{ns}$  is  $\text{kg}^{1-n} \text{ grain ha}^n \text{ kg}^{-1}$  nutrient. Hence, if  $n = 0$ , the dimensions of  $\varepsilon_{s,\theta}$  and  $\varepsilon_{ns}$  are the same. Furthermore, the power constant,  $n$ , is expected to be larger than zero, that is,  $\varepsilon_{s,\theta}$  is expected to be an increasing function of  $\theta$ .

With the use of Equations (2) and (3) it follows that  $\varepsilon_s$  and  $\varepsilon_f$  would be proportional to rainfall:

$$\varepsilon_{ns}, \varepsilon_{nf} \propto (\Delta r)^n$$

where values of  $n$  would be in the range of 1.4 ( $GY_x$ ) to 1.9 ( $GY_{\theta}$ ) for phosphorus; cf. Table 5. In a related study, a coefficient of  $n = 1.5$  was found to give the best fit for phosphorus, based on a large and reliable dataset (Harmsen, 1995).

Hence, the availability of P is strongly affected by moisture and, in particular at very low moisture contents, the availability is very low (Figure 3). Such a type of behaviour could be explained by assuming that P in the soil reaches the plant roots largely by diffusion through the liquid phase and that the ratio of the 'effective' diffusion coefficient,  $D_p$ , to the molecular diffusion coefficient,  $D$  ( $\text{cm}^2 \text{ sec}^{-1}$ ), is proportional to the moisture content of the soil,  $\theta_s$ , according to (Olsen & Kemper, 1968):



$$D_p/D \propto a\theta_s + b\theta_s^2$$

where  $a$  and  $b$  are constants related to the properties of the soil. Over a limited range of moisture contents this relation can be approximated by:

$$D_p/D \propto \theta_s^n$$

with  $n > 1$ . Hence, the fact that the availability of soil phosphorus tends to increase with increasing rainfall, may well be related to the diffusion behaviour of dissolved phosphorous from reactive surfaces to plant roots in the soil. Therefore, the diffusion behaviour of phosphorus in soils and the limited availability of phosphorus at low moisture contents may well explain the relatively large response to P-fertilizer application under low rainfall conditions (Matar *et al.*, 1992).

In the above discussion of the availability of phosphorus in soil, no distinction is made between native soil phosphorus (e.g., P in unfertilized soil), fertilizer phosphorus, and residual phosphorus, i.e., phosphorus derived from P-fertilizer application in previous years. In principle, the diffusion behaviour of all three forms of phosphorus would be affected by soil moisture in a similar fashion, but at the same time the chemical and physico-chemical interactions as well as the spatial distribution of the different forms of phosphorus may be quite different and thus affect availability and phosphorus-use efficiency differently. In a related study, such differences were indeed found (Harmsen, 1995), but the present dataset from the NxP trials does not allow to distinguish between different forms of phosphorus with sufficient accuracy.

Assuming that Equation (6) also applies to nitrogen, it follows that  $\epsilon_{ns}$  and  $\epsilon_{nf}$  would be proportional to rainfall with values of  $n$  in the range of 0.3 ( $GY_x$ ) to 0.6 ( $GY_0$ ); cf. Table 5. The increased availability of total mineral nitrogen (cf. Figure 3) is assumed to be, at least partly, due to increased accessibility of mineral nitrogen in soil, because of increased root development and activity, and access to a larger soil volume with increasing soil moisture. However, the apparent increase in availability of soil mineral nitrogen could also be due to net mineralization of soil organic nitrogen. In studies on the mineralization of nitrogen in soils of the rainfed agricultural region of Syria it was observed that the onset of the rains was followed by a flush in biological activity in the soil, resulting in net immobilization of mineral nitrogen. This phase was followed in late winter and early spring by a phase of net mineralization of organic nitrogen. The mineralization process was found to be moisture-dependent (Harmsen, 1984 & 1987; Buresh *et al.*, 1990).

In the present treatment, no distinction is made between the availability of soil mineral nitrogen and fertilizer-derived nitrogen. The reference to the increased availability of nitrogen through net mineralization applies to soil organic nitrogen only and not to fertilizer nitrogen, although some fertilizer nitrogen that is immobilized early in the season may be mineralized in early spring and become available to the crop (Harmsen, 1987). Fertilizer nitrogen is subject to loss mechanisms, such as ammonia volatilization from urea or ammonium-fertilizers, which do not, or to a lesser extent affect soil mineral nitrogen. In fact, one would not expect the availability of fertilizer-N, once it has entered the soil, to be much affected by soil: at higher rain-

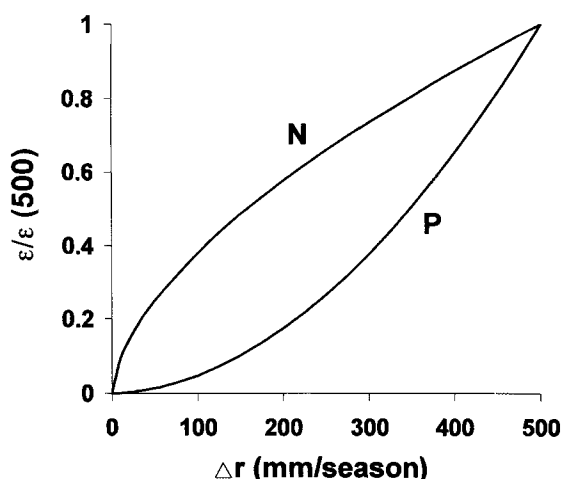


Figure 3. The water-dependent nutrient availabilities according to equation (5), with the use of Equations (2) and (3), for nitrogen ( $n = 0.6$ ) and phosphorus ( $n = 1.9$ ) plotted as their relative values,  $\varepsilon_{s,\theta}(\Delta r)/\varepsilon_{s,\theta}(\Delta r = 500)$ , against  $\Delta r$  ( $\text{mm year}^{-1}$ ).

fall conditions leaching may increase, but this may be compensated for to some extent by more prolific root systems and deeper rooting. Therefore, the apparent moisture dependence of soil and fertilizer nitrogen is presumably mainly an effect increased availability of soil mineral nitrogen, but may include the effect of net mineralization of soil organic nitrogen. The latter effect should, strictly speaking, be included in the modified Mitscherlich equation through the introduction of  $N_s = N_{s,\theta}$ , rather than through  $\varepsilon_{s,\theta}$ . However, the N×P dataset does not allow to distinguish between these mechanisms with sufficient accuracy and therefore the effects of rainfall on the nutrient part of the exponential function have all been pooled in the factor  $GY_\theta^{n-1}$  in the Mitscherlich equation. However, if more accurate or detailed information would be available, then it would be desirable to refine the treatment.

The relation between nutrient availability and rainfall can also be estimated directly from the modified Mitscherlich equation, for a specified yield level. In Figure 4 these relations are given for soil phosphorus (Figure 4a) and soil mineral nitrogen (Figure 4b). For example, for soil phosphorus and a yield level of  $GY = 0.8GY_\theta$ ,  $N_s$  would decrease from 9.9 ppm at 200 mm season<sup>-1</sup> to 2.1 ppm at 600 mm season<sup>-1</sup>, whereas for  $GY = 0.9GY_\theta$ ,  $N_s$  would decrease from 14.2 ppm at 200 mm season<sup>-1</sup> to 3.0 ppm at 600 mm season<sup>-1</sup>. Although the dataset of the N×P trials does not allow to estimate  $\varepsilon_{ns}$  with sufficient accuracy, the order of magnitude and the trend are clearly in agreement with observations in field experiments in Syria (e.g., Matar *et al.*, 1992; Harmsen, 1995). At higher rainfall levels, values of P-Olsen below 5 ppm may give no or a very limited response, whereas at lower rainfall levels, values of P-Olsen of 7 ppm or more may give clear yield responses to applied phosphorus.

In the case of nitrogen (Figure 4b), the trends also seem to be in agreement with field observations: At very low moisture contents the nitrogen is relatively unavailable, but with increasing rainfall the availability increases. An alternative explanation would be that at low moisture contents mineralization does not occur, whereas with increasing moisture net mineralization contributes to the crop's nitrogen re-

# CASE STUDY OF A MODIFIED MITSCHERLICH EQUATION

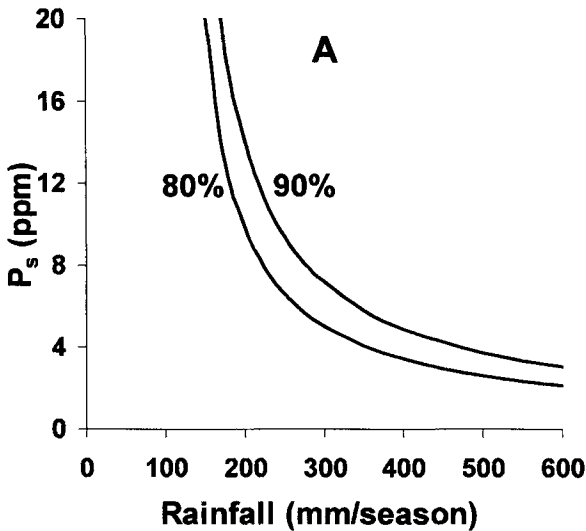
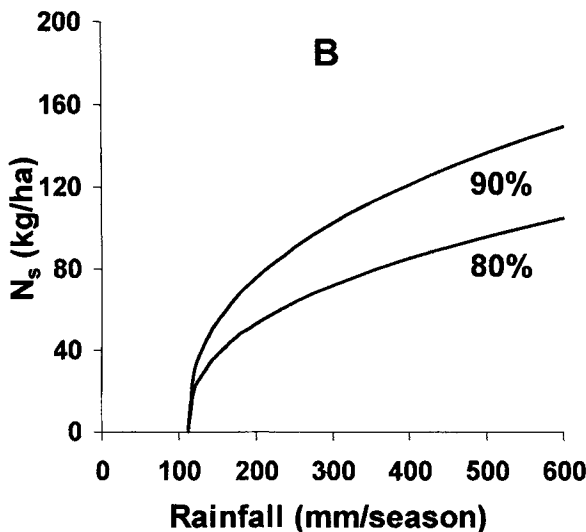


Figure 4. The availabilities of (A) soil phosphorus (Equation 4 for  $n-1 = 0.9$  and  $P_f = 0$ ) and (B) soil mineral nitrogen ( $n-1 = -0.4$  and  $N_f = 0$ ) for two values of  $GY$ :  $GY = 0.8GY_0$  (80%) and  $GY = 0.9GY_0$  (90%), plotted against seasonal rainfall (mm year<sup>-1</sup>).



quirement. The data in Figure 4 would indicate that at 600 mm season<sup>-1</sup>, that is, at grain yields of 5466 kg (80%) or 6149 (90%), the quantities of soil mineral nitrogen available for the entire crop (including straw) would be 105 and 150 kg N ha<sup>-1</sup>, respectively. Assuming that about 70% of the total crop nitrogen is in the grain, this would result in 1.34% N (80%) and 1.71% N (90%) in the grain. In related studies in Syria, about 2% N was found in the grain at levels of fertilizer application that were considered sufficient for near-potential growth (Harmsen *et al.*, 1983). Hence, levels

of 1.34 and 1.71, respectively, are not very different from what would be expected on the basis of related research in Syria, and the trend is in agreement with observations in fertilizer experiments in Syria (Anderson, 1985a,b; Harmsen, 1987, 1995). Although the estimates of  $\epsilon_{ns}$  for N and P could possibly be improved, the results presented here illustrate how the modified Mitscherlich equation could be used as a basis for fertilizer recommendations.

### *Nutrient-use efficiency*

The nutrient-use efficiencies by rainfed crops can be obtained by taking the partial derivatives in Equation (4). For soil nutrients ( $N_s$ ) the partial derivative becomes:

$$\partial GY / \partial N_s = \epsilon_{ns} (GY_\theta - GY) GY_\theta^{n-1} \quad (6)$$

and for fertilizer nutrients ( $N_f$ ):

$$\partial GY / \partial N_f = \epsilon_{nf} (GY_\theta - GY) GY_\theta^{n-1} \quad (7)$$

The phosphorus-use efficiencies by rainfed crops can be obtained by taking  $n-1 = 0.9$  in Equations (6) and (7), from which it follows that the phosphorus-use efficiency is assumed to increase quite steeply with increasing rainfall. Also, if  $GY = 0$ , the phosphorus-use efficiency (or slope of the yield response curve) would be proportional to  $\epsilon_{nf} GY_\theta^{1.9}$ . This could be characterized as Mitscherlich-Liebscher-plus behaviour, that is, the phosphorus-use efficiency at  $GY = 0$  increases more than linearly (almost quadratically) with  $GY_\theta$  (cf. De Vries, 1939; De Wit, 1992, 1994).

The nitrogen-use efficiencies follow from Equations (6) and (7) by taking  $n-1 = -0.4$  (see Table 5). Hence, the nitrogen-use efficiency is also assumed to increase with increasing rainfall, but much less so than in the case of phosphorus.

If  $GY = 0$ , the nitrogen-use efficiency (or slope of the yield response curve) would be proportional to  $\epsilon_{nf} GY_\theta^{0.6}$ . This behaviour would be intermediate between Liebig and Mitscherlich-Liebscher (Harmsen, 2000), that is, the nitrogen-use efficiency at  $GY = 0$  increases roughly with the square root of  $GY_\theta$ .

### *Conclusion*

The results of field experiments presented in this paper support the assumption that water-limited potential yield in the semi-arid areas can be described, above a certain threshold level, as an approximately linear function of rainfall. An evaluation of the modified Mitscherlich equation, using a dataset from rainfed cereal production in the semi-arid areas of Syria, showed that rainfall explained most of the variation in yield between locations and seasons, and that the moisture-dependent nutrient part in the Mitscherlich equation explained a small but significant part of the yield variation as well. The model discussed in this paper could provide a framework for a Mitscherlich-type approach to describing crop response to nutrient availability c.q. fertilizer application under rainfed conditions in semi-arid regions.

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